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MANEL ANTELO

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Monopoly, asymmetric information, and optimal environmental taxation*

Manel Antelo**
Universidad de Santiago de Compostela

RESUMEN
Este artículo analiza el nivel óptimo de impuestos ambientales en un modelo en el que un monopolista contaminador produce a lo largo de dos periodos. Suponemos que los costes de producción del monopolista sólo son conocidos por él y que el planificador, que puede inferir dichos costes observando el output que produce el monopolista durante el primer periodo, tiene la posibilidad de fijar impuestos ambientales que afectan a la emisiones; además, el monopolista puede elegir un nivel de output durante el primer periodo distinto del nivel óptimo al objeto de manipular las creencias del regulador. En este contexto, si el regulador valora mucho la calidad ambiental, el monopolista de coste bajo tiene incentivos a camuflarse por una empresa de coste alto para lograr que el impuesto del segundo periodo sea de menor cuantía. Esto lleva a la empresa de coste alto a producir, en el primer periodo, un nivel de output menor o igual que el correspondiente a la maximización miope de beneficios. El impuesto óptimo en el primer periodo cuando el output de la empresa señaliza el coste de producción es, pues, menor o igual que el que habría si el output no señalizase el coste. El nivel esperado de contaminación en el primer caso es también menor o igual que en el segundo. Por el contrario, cuando el regulador valora poco el medio ambiente, el impuesto ambiental se vuelve negativo (un subsidio por unidad de polución) y de mayor o igual cuantía en el contexto de señalización que en el contexto de no señalización.

Palabras clave: Política de impuestos y subvenciones ambientales, empresa monopolista contaminante, información asimétrica vertical, señalización y no señalización

ABSTRACT
This paper aims to examine optimal environmental taxation in an incomplete-information two-period model in which a monopolistic firm produces and pollutes. It is assumed that the polluting firm is privately informed about its costs of production, and the policymaker, which can only infer the firm's costs from observing the output produced in

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** Departamento de Fundamentos da Análise Económica, Universidade de Santiago de Compostela, Campus Norte, 15782 Santiago de Compostela, Spain (*e-mail:* aepantel@usc.es).

the first period, has the chance to set environmental taxes to affect emissions; the emitter of pollution may then choose a non-optimal level of production in such a period in order to manipulate the policymaker's beliefs concerning its costs. If the policymaker values environmental quality sufficiently, the low-cost polluter has an incentive to misrepresent itself as a high-cost firm in order to secure a low environmental tax in the second period. This leads the high-cost polluting firm to produce, in the first period, an output level that is not higher than output which would be optimal if only short-term considerations were taken into account. The optimal environmental tax rate in the first period, when the firm's output is a signal of its cost, is then lower than or equal to what it would be if the firm's output was not a signal of firm's costs. The expected emissions in the former context are also lower than or equal to those in the latter case. By contrast, when the policymaker's valuation of the environment is sufficiently low, the environmental tax is negative (a subsidy per unit of pollutant emitted) in both the signaling and non-signaling contexts and no less in the former context than in the latter.

Keywords: Environmental tax and subsidy policy, monopolistic polluting firm, vertical asymmetric information, signaling and non-signaling
JEL classification: D62, D82, L13

1. Introduction

Many activities of production and consumption have significant environmental effects because of the widespread use of fossil fuels and other environmentally harmful substances. The size of such effects is of concern not only because of their impact on climate change but also because they influence other external costs. Economists have paid increasing attention to the question of whether it is possible to use economic instruments to improve the quality of the environment and, in particular, to mitigate the effects of emissions on climate change.

Recent trends in many countries indicate that a wide range of incentive and information-based public policy instruments have been developed for the purpose of environmental regulation (OECD, 2003). Traditional command-and-control regulation systems that impose emission standards have been criticized, in theoretical terms, for being inefficient. However, the fact that environmental taxes rely on a price mechanism rather than the administrative prices associated with command-and-control systems increases their efficiency and lowers overall compliance costs. In parallel, it has also been argued that environmental taxes produce the greatest reduction in pollution because firms attempt to reduce their costs, where necessary by introducing new technologies.

The question of environmental taxes and their environmental effectiveness have been almost above criticism from an academic viewpoint since they were first proposed by Pigou (1920), who argued that taxes on emissions would reduce pollution in the most efficient way possible with virtually no distortion of the economy.[1] The idea that taxing an undesirable activity such as pollution makes much more sense than taxing goods such as income, work or savings has been introduced in both the academic and political spheres. This concept of taxing unwanted activities was given impetus in the early 1990s when a number of economists suggested the so-called "double dividend" theory.[2]

In addition to the importance of environmental taxes in the theoretical agenda, in practice environment-related taxes are growing in importance across the industrial world. As indicated by OECD data (OECD, 2003), such taxes represented, in 1998, around 3 percent of GDP and raised 7 percent of all tax revenues in the 21 OECD countries in 2000. South Korea was at the top of this league, with environmental taxes accounting for almost 12 percent of total tax revenues —

[1] Actually, the Pigouvian tax, set at the marginal damage, is efficient only if the economy is otherwise undistorted.

[2] The "double dividend" theory will not be discussed in this paper. For a review on this subject, see, for example, Bovenberg (1999) and Schöb (2003).

although consisting entirely of taxes on oil products and vehicles. The OECD figures for other countries show that Norway, the UK and Denmark raise more than the average level of revenue through environment-related taxes.

The OECD data also indicate that environmental tax revenues rose during the 1990s, partly because new taxes on waste and energy were introduced in many member countries. Several European countries and Japan tax power generation; Scandinavian countries and the Netherlands levy carbon dioxide taxes; Scandinavian countries, Denmark, Switzerland, France, Italy, the Czech Republic, Spain (region of Galicia), and Korea levy sulphur taxes on oil products and electricity generation; Sweden and France apply a nitrogen oxide tax; Denmark and Norway levy a tax on chlorinated solvents; Italy, the US and the Czech Republic are among countries which tax lubricant oils; Belgium, Canada and Sweden are among those which tax pesticides; and Sweden and the US tax commercial fertilizers.

Finally, other prominent example is the US, where the Environmental Protection Agency reports that the number of economic mechanisms to reduce pollution rose from 40 in 1992 to at least 100 in 1997. Many taxes are levied at the state level, such as a litter tax in Washington State, a fertilizer tax in Iowa and a landfill tax in Massachusetts.

Direct taxation on pollution and indirect environmental levies that have been introduced in many developed countries indicate a rising demand for a cleaner environment coupled with growing fiscal pressures. Although more recent research indicates that the cost of a cleaner environment may be somewhat higher than previously indicated by the “double-dividend” hypothesis,[3] two conclusions hold: environment-related taxes still look like the least costly way of getting a clean environment,[4] and they are becoming an increasingly important part of the fiscal systems of OECD countries (Morgenstern, 1995).

The question of the level at which pollution taxes must be set has been explored to a great extent in the literature. Most models have assumed that all parties have perfect information about the technology of production and any other issue relevant to the polluters. However, less attention

[3] The “double-dividend” theory was reframed by the more recent literature, which emphasizes the fact that environmental taxes may cause potentially significant distortions in the factors and product markets that undermine the “double-dividend” hypothesis, although virtually *any* environmental regulatory instrument (including taxes, regulations, and tradable emissions permits) tends to compound pre-existing distortions in the tax system — a cost that is recognized as “tax interactions” or “interdependency effects”. See Morgenstern (1995) for a discussion on this subject.

[4] Cairncross (1995) points out that countries that used environmental taxes have found them to be effective. For instance, when Sweden introduced in 1992 a tax of $6000 per ton on nitrous oxide emissions from power plants, average emissions fell by 35 percent in two years. Likewise, the Swedish sulphur tax, which was introduced in 1991, may be estimated to be responsible for 30 percent of the total reduction in sulphur emissions from 1989 to 1995 (SEPA, 1997).

has been paid to the case where there is incomplete information. On the assumption that the optimal strategy for pollution taxes may be very sensitive to the information structure assumed, the focus of the current paper is the case where the policymaker has imperfect information.[5] That is, polluters are assumed to have an informational advantage concerning the technology of production they currently use (i.e., concerning their emissions) with respect to the policymaker. This source of informational asymmetry is not only a major difficulty in the policymaking arena, but it may also lead firms to have, in a dynamic setting, the potential and the incentive to exploit their advantage with the aim of undermining the intended goals of external parties such as the policymaker. After observing the firm's actions in a given period, the policymaker has the potential to adjust its regulatory action for the next period. This may produce a conflict between the welfare and informational objectives that may significantly affect the optimal level of pollution taxes.

When polluters have an informational advantage over third parties and they are large enough to have an effect on market and regulatory actions, it would seem appropriate to examine the regulator's behavior in a dynamic context in which polluters act strategically. The aim of the current paper is to explore the extent to which the existence of vertical asymmetric information concerning the costs of polluting firms, and the possibility of signaling, affect the optimal level of pollution taxes in a dynamic context. In the model, we consider the case of a single polluter that may be of a low-cost or high-cost type. The firm and the policymaker are engaged in a two-period incomplete information game with the following timing. In period 1, the policymaker, without knowledge of the costs of the polluter, announces and commits to a per-unit environmental tax for this period, and the polluter then produces an output level which generates a given level of emissions. In period 2, the policymaker may use the level of output from the polluter to draw conclusions about the cost structure of the firm. The policymaker then sets a per unit output environmental tax. In the current paper two scenarios are examined: where the policymaker uses the output levels in period 1 to set the level of taxation in period 2 (the signaling context), and where the policymaker does not use these observations when setting the level of taxation in period 2 (the non-signaling context). Finally, the polluter chooses the output level for period 2.

If we consider a monopolist whose production damages the environment, two types of distortions exist: the distortion due to the environmental damage and the distortion of the firm's underproduction linked to the exercise of market power. As a consequence, the policymaker, when designing the optimal environmental tax in the non-signaling regime, simply balances the

[5] Barigozzi and Villeneuve (2004) focus on the signaling effect in tax policy when agents are less informed about the effect of their consumption than the policymaker. In this setting, they show that optimal taxes in a symmetric information context are cannot be implemented under asymmetric information.

production and the environmental damage. In such a trade-off, and although the monopolist's expected output is lower than the socially efficient level and any output tax reduces its production further, a positive environmental tax is optimal when the policymaker's valuation of the environment is high enough.

In a signaling context, when selecting the optimal environmental tax for period 1, we must add two additional effects to be considered by the policymaker: The indirect- or strategic-signaling effect and the direct-signaling effect. The indirect-signaling effect works as follows: if the valuation of the environment is sufficiently high, a polluter with low costs has an incentive to misrepresent itself as a high-cost firm, with the aim of producing a lower environmental tax in period 2. This is understood by the policymaker and induces the policymaker to commit to a lower environmental tax in period 1 than it would have in the absence of signaling. In fact, in order to outweigh the incentive for a low-cost polluting firm to misrepresent itself as being high-cost, the policymaker has to make downward distortions of period 1 output more costly, which is achieved by reducing the amount of the environmental tax. The direct-signaling effect encourages the polluter to produce a lower output in period 1 than in the non-signaling context. This effect also produces pressure on the policymaker to set a lower environmental tax in the signaling context than in the non-signaling context.

In the following sections the two-period environmental tax game is examined to find the equilibria. When the high-cost and low-cost polluting firms produce a different output at t=2, this is referred to as a separating sequential equilibrium. By contrast, when high-cost and low-cost firms produce the same output at t=2, this is referred to as a pooling sequential equilibrium.

At deriving the separating sequential equilibrium of this two-period signaling game, it is shown that a high-cost polluter, to convince the policymaker that it is of a high-cost type, needs to distort its period 1 output below the profit-maximizing level as a monopolist. As a result, signaling significantly decreases the optimal period 1 environmental tax rate relative to the case in which signaling is absent. When the firm's output of period 1 equals the profit-maximizing level as a monopolist, signaling does not affect the optimal period 1 environmental tax rate with respect to the non-signaling context.

In the model, the case of a negative environmental tax (a subsidy per unit of pollutant emitted) also emerges in both contexts when the policymaker's valuation of the environmental damage is sufficiently low. In these circumstances, the market distortion due to underproduction is much stronger than the distortion due to the environmental damage caused by polluting emissions and, thus, there is a subsidy per unit of pollutant emitted. Furthermore, in the signaling context, the polluter has an incentive to be perceived by the regulator as a low-cost firm, which may lead the low-cost firm to increase its output level of period 1 (direct-signaling effect). The policymaker is thus induced in this way to increase the per-output subsidy with respect to the non-signaling context. In addition, and to reinforce the incentive of the high-cost firm to misrepresent itself as a low-cost firm, the policymaker has to make the upward distortions of period 1 output less costly (strategic-signaling effect). This is achieved by increasing the amount of the subsidy per unit of pollutant emitted. Summing up, these two effects reinforce each other and the environmental subsidy per unit of pollutant emitted in the signaling case is greater than or equal to the subsidy in the non-signaling case.

The remainder of the paper is made up of six sections. In Section 2 the model is outlined. The separating sequential equilibrium of the signaling game is derived in Section 3, together with the optimal period 1 and period 2 emission taxes. Section 4 discusses the pooling sequential equilibrium and the environmental tax in a non-signaling regime. In Section 5, the signaling outcome and the non-signaling outcome are compared. Section 6 is devoted to the case in which the policymaker's valuation of the environmental damage is sufficiently low. Conclusions are presented in Section 7.

2. The model

Consider a single industry constituted by a firm which is the sole producer for two periods, indexed by t=1,2.[6] In each period, the inverse demand function for the good produced by such firm is assumed linear and, without further loss of generality, of the form

$$p_t(q_t) = 1 - q_t\,, \tag{1}$$

where p_t denotes the unit price in period t when q_t units of output are sold in this period.[7] The absolute size of the market is normalized at one and the inverse demand function remains unchanged from one period to the other.

In addition, assume that the firm's production results in emissions that damage the environment. The emissions increase with the level of output. Particularly, and following Ulph (1996), each unit of the good produced causes one unit of polluting emissions in such a way that

[6] The consideration of a monopolistic polluting firm tries to reflect the fact that companies in many polluting industries as, for instance, power stations are mostly large firms with market power.

[7] This demand comes from the maximization problem of a representative consumer with utility separable in money, m_t, given by $U_t(q_t) = u_t(q_t) + m_t$, where $u_t(q_t) = (1 - q_t/2)\,q_t$ is the utility function of the consumption good.

the environmental damage is measured, in each period, by the convex function of the pollution level

$$ED_t(q_t) = (d/2)q_t^2, \tag{2}$$

where $d > 0$ is an exogenous parameter that represents the valuation of the environment made by the policymaker or, alternatively, that measures the degree of ecological conscience.[8]

The marginal production cost of the polluting firm is constant and can take either a low or a high value at random. Specifically,

$$\tilde{c} = \begin{cases} 0 & \text{with probability } \gamma \\ c & \text{with probability } 1-\gamma, \end{cases} \tag{3}$$

where $\gamma \in (0,1)$ is taken as exogenous and technological parameter c, representing the efficiency gap between the high-cost firm and the low-cost firm, is assumed to satisfy $0 < c < 2/(d+1)$ given the inverse demand defined in (1) and the pollution level function stated in (2).

The policymaker uses the environmental tax the polluting firm has to pay per unit of pollution emitted as a decision variable to control the emissions in each production period.[9] It is further assumed that both the polluting firm and the policymaker are risk-neutral, and that the discount factor between periods is one.

The environmental-tax game involves four stages. At the beginning of period 1 (first stage) and before observing the output choice of the firm, the policymaker acts as a Stackelberg leader in setting the environmental tax for period 1, e_1. The policymaker has worse information than the firm itself concerning the environment damage that it causes. The only thing which is common knowledge at this date is the distribution of the firm's cost. Given the prior probability assessment that the polluter is a low-cost firm and the environmental tax chosen by the policymaker in period 1, in the second stage of the game the monopolist acts as a Stackelberg follower in deciding the profit-maximizing output for period 1, q_1. It is assumed that, at the end of this period, the monopolist's output during period 1 is observed by the policymaker, from which its probability assessment regarding the monopolist's marginal cost is updated. Let $\gamma(q_1)$ be the common updated probability assessment as to the likelihood of the polluting firm being of a low-cost type.

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[8] Such a parameter may also be interpreted as the marginal willingness to pay for decreasing the environmental damage by one unit.

[9] In reality, it is a tax on output not emissions, but the same result holds for emissions taxes.

Next, at the beginning of period 2 and given the updated beliefs of the policymaker formed after observing the firm's output in period 1, the policymaker announces and commits in the third stage to a period 2, per unit output environmental tax, e_2. Finally, in the fourth stage of the game, the polluter chooses the profit-maximizing output in period 2, q_2, given the updated probability assessment $\gamma(q_1)$ and the environmental tax e_2 for the period.

As mentioned above, in each production period the policymaker asks for a per unit charge on emissions so as to maximize the total social surplus per period.[10] This is composed of the unweighted sum of the consumer surplus, the firm's profit and the governmental revenue from the pollution tax[11] minus the environmental damage caused by the total pollution due to firm's production. Namely,

$$W_t(q_t) = CS_t(q_t) + \Pi_t(q_t) + T_t(q_t) - ED_t(q_t)$$
$$= ((1-d)/2)q_t^2 + q_t^2 + e_t q_t . \qquad (4)$$

The equilibrium concept used for solving the proposed environmental-tax game is the sequential equilibrium (Kreps and Wilson, 1982), in which the period 1 output must constitute a Bayesian-Cournot equilibrium, the period 2 output must be chosen optimally given the updated probability assessments, and beliefs must satisfy the Bayes' rule (when it applies). In order to examine the role played by the information transmission on optimal environmental taxation, two types of sequential equilibria are considered: separating and pooling equilibria.[12]

3. Separating sequential equilibria

In a separating sequential equilibrium, period 1 output of the pollutant firm conveys full information concerning firm's costs, by which the game of period 2 becomes a complete information game in which the policymaker is fully informed about the firm's marginal cost. A

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[10] Administrative costs associated with the environmental taxes are considered to be negligible. This assumption is in accordance with the conclusions drawn, for instance, for the Swedish Environment Protection Agency (see SEPA, 1997, p. 45).

[11] Policymaker's revenues from pollution tax may be understood as an amount that is to the advantage of the community in the form of social expenditure or that may serve to alleviate some of the distorting taxes in the economy.

[12] Since the main purpose of the paper is to examine the role of signaling and the subsequent transmission of information on pollution taxes compared with the case in which signaling is absent, neither hybrid nor semi-separating equilibria are considered.

separating sequential equilibrium is, in this setting, a list of actions and beliefs $\{e_1(\tilde{c}), q_1(\tilde{c}, e_1), \gamma(q_1(\cdot)), e_2(\tilde{c}, q_1(\cdot)), q_2(\tilde{c}, e_2, q_1(\cdot))\}$ that adopts the form

$$e_1(\tilde{c}) = e_1, \text{ for all } \tilde{c} \in \{0, c\}, \tag{5}$$

$$q_1(\tilde{c}, e_1) = \begin{cases} q_{1H}, & \text{if } \tilde{c} = c \\ q_{1L}, & \text{if } \tilde{c} = 0, \end{cases} \tag{6}$$

$$\gamma(q_1) = \begin{cases} 0, & \text{if } q_1 = q_{1H} \\ 1, & \text{if } q_1 = q_{1L}, \end{cases} \tag{7}$$

$$e_2(\tilde{c}, q_1(\cdot)) = \begin{cases} e_{2H}, & \text{if } q_1 = q_{1H} \\ e_{2L}, & \text{if } q_1 = q_{1L}, \end{cases} \tag{8}$$

and

$$q_2(\tilde{c}, e_2, q_1(\cdot)) = \begin{cases} q_{2H}^m, & \text{if } \tilde{c} = c \text{ and } q_1 = q_{1H} \\ q_{2L}^m, & \text{if } \tilde{c} = 0 \text{ and } q_1 = q_{1L}, \end{cases} \tag{9}$$

where subscripts *H* and *L* denote the high-cost and the low-cost firm, respectively. That is, the policymaker selects a period 1 environmental tax, e_1, given the probability assessment of the polluting firm being of low-cost, and the polluter chooses a period 1 output level, q_1, given the environmental tax to be paid. Next, for every q_1 quoted, the policymaker forms an updated belief about the type of polluting firm and sets the optimal environmental tax for period 2, e_2, and the polluter selects its output level, q_2. The updated beliefs $\gamma(q_1)$ are unrestricted, except that Bayes' rule is used to form them for actions with positive probability in equilibrium. As usual, a separating equilibrium such as that defined in (5)-(9) is determined by working backwards from the second period to the first.

Period 2

Given the profit function of the polluter in period 2, $\Pi_2 = (1 - \tilde{c} - q_2)q_2 - e_2 q_2$, its optimal production level in such a period is given by[13]

$$q_2 = (1 - \tilde{c} - e_2)/2, \ \tilde{c} \in \{0, c\}, \tag{10}$$

and the objective of the policymaker is to find the level of environmental tax that maximizes the social welfare at *t*=2. The resolution of the problem

$$\max_{e_2} W_2 = ((1-d)/2)\left(\frac{1-\tilde{c}-e_2}{2}\right)^2 + \left(\frac{1-\tilde{c}-e_2}{2}\right)^2 + e_2\left(\frac{1-\tilde{c}-e_2}{2}\right) \tag{11}$$

yields the optimal environmental tax

$$e_2 = (d-1)(1-\tilde{c})/(d+1), \tag{12}$$

where it can be observed that the lower the firm's marginal cost the greater the environmental tax at period 2, whenever $d > 1$. In this case, the policymaker places a high value on the environment, and the more (output and) pollutant emitted the higher the per unit output environmental tax. This leads the polluter to have an incentive to be perceived by the policymaker as a high-cost firm in period 2, in order to bear a lower environmental tax in the period. The contrary holds when $d < 1$, in which case the environmental tax for period 2 becomes negative (a subsidy per unit of pollutant emitted) and higher when the firm is low-cost than when it is high-cost. Thus, the monopolistic firm has an incentive to be perceived as a low-cost firm by the policymaker.

Substituting the result from (12) into (10) affords both the optimal output produced by the monopolist (and the pollutant emitted) $q_2 = (1-\tilde{c})/(d+1)$ in period 2, by which its maximized profit in this period is

$$\Pi_2 = ((1-\tilde{c})/(d+1))^2. \tag{13}$$

[13] Both second-order conditions for maxima and stability conditions are fulfilled.

Denote by $\Pi_{2L}(q_{2L} \mid \gamma(\cdot)=1)$ the maximized profit of the low-cost monopolist in period 2 when it signaled itself as a low-cost firm, and by $\Pi_{2L}(q_{2L} \mid \gamma(\cdot)=0)$ the maximized profit of the low-cost monopolist in period 2 when it convinced the policymaker that it is a high-cost firm. Similarly, let $\Pi_{2H}(q_{2H} \mid \gamma(\cdot)=0)$ be the maximized profit of the firm in period 2 when it is of high-cost and signaled itself as such and $\Pi_{2H}(q_{2H} \mid \gamma(\cdot)=1)$ the firm's maximized profit of period 2 when it is of high-cost type but the policymaker believes it is a low-cost firm. The following lemma summarizes the profits obtained by the polluter in the second period when it is honest and reveals its true type compared to the case in which it misrepresents itself as another type.

Lemma 1. *The emitter's profits in period 2 are as follows:*

(i) *If* $d>1$, *then* $\Pi_{2L}(q_{2L} \mid \gamma(\cdot)=0) > \Pi_{2L}(q_{2L} \mid \gamma(\cdot)=1)$, *when the polluter is a low-cost firm, and* $\Pi_{2H}(q_{2H} \mid \gamma(\cdot)=0) > \Pi_{2H}(q_{2H} \mid \gamma(\cdot)=1)$, *when it is of high-cost type.*

(ii) *If* $d<1$, *the opposite holds.*

Proof. See the Appendix.

The lemma states that, regardless of its true cost, the pollutant firm has an incentive to be perceived by the policymaker as a high-cost firm in period 2, whenever the policymaker's valuation of the environmental damage is high enough. This is due to the fact that in this case the policymaker believes that the polluting firm generates a lower pollution level and, consequently, the polluting firm is taxed with a lower environmental tax in such a period.

Period 1

In this period, the firm's marginal cost is private information. Given that the period 1 environmental tax is announced and committed before the polluting firm produces, it is necessary to determine the optimal period 1 output of the firm for any pollution tax. In a separating sequential equilibrium, the polluting firm signals each one of its possible marginal costs by selecting a different output in period 1. Thus, it seems reasonable to assume that the signal q_{1H} sent by the high-cost polluting firm is associated with updated belief $\gamma(q_{1H})=0$ and the signal q_{1L} of a low-cost firm with posterior belief $\gamma(q_{1L})=1$. In addition, and to restrict the out-of-equilibrium updated beliefs, it suffices that $\gamma(z_1)=1$ for any other output level z_1 satisfying $z_1 \notin \{q_{1H}, q_{1L}\}$.[14]

Denoting by superscript *m* the monopoly regime, it is clear that to form part of a separating equilibrium the output produced in period 1 must satisfy the incentive compatibility conditions

$$q_{1L} = q_{1L}^m , \tag{14}$$

$$\Pi_{1H}^m - \Pi_{1H}(q_{1H}) \leq \Pi_{2H}(q_{2H} \mid \gamma(q_{1H}) = 0) - \Pi_{2H}(q_{2H} \mid \gamma(q_{1H}^m) = 1) , \tag{15}$$

and

$$\Pi_{1L}^m - \Pi_{1L}(q_{1H}) \geq \Pi_{2L}(q_{2L} \mid \gamma(q_{1L}^m) = 0) - \Pi_{2L}(q_{2L} \mid \gamma(q_{1H}) = 1) . \tag{16}$$

Succinctly, condition (14) indicates that the best a low-cost polluting firm can do in period 1 is to produce the profit-maximizing output as a simple monopolist. Inequality (15) is the incentive compatibility condition for a high-cost polluting firm. It states that such a firm would prefer to produce output q_{1H} in period 1 (an output that may or may not differ from the profit-maximizing output as a simple monopolist q_{1H}^m) and convince the policymaker that it is a high-cost firm, rather than produce the output that maximizes its profit in period 1 as a high-cost monopolist and then be perceived as a low-cost firm. Finally, condition (16) is the self-selection constraint for a low-cost polluting firm. It establishes that such a firm would prefer to produce output q_{1L}^m in period 1 and be perceived as a low-cost firm, rather than be perceived as a high-cost firm by being obliged to produce output q_{1H} in period 1. The resolution of these conditions enables us to obtain the period 1 outputs which form part of a separating sequential equilibrium.

Lemma 2. *When $d > 1$, the output produced by the emitter at t=1 that forms part of a separating sequential equilibrium is as follows:*

(i) $q_{1H} = (1-e_1)/2 - \sqrt{(d-1)[4+(d-1)c]\,c}\,/2(d+1)$ *when the firm is of high-cost type, and* $q_{1L} = (1-e_1)/2 \equiv q_{1L}^m$ *when it is of low-cost, if parameters d and c satisfy* $(d+1)c - \sqrt{(d-1)[4+(d-1)c]\,c} < 0$ *;*

[14] This restriction on updated beliefs punishes any deviation from the equilibrium path as much as possible.

(ii) $q_{1H} = (1-c-e_1)/2 \equiv q_{1H}^m$ *when the firm is of high-cost type and* $q_{1L} = (1-e_1)/2 \equiv q_{1L}^m$ *when it is a low-cost firm, if parameters d and c do not satisfy the inequality.*

Proof. See the Appendix.

The incentive to mislead the policymaker about its marginal cost leads the high-cost polluting firm — in order to distinguish itself from the firm of low-cost type — to reduce its output of period 1 below the level that would maximize its profits as a simple monopolist in such a period, when its marginal cost is not very high. Conversely, the low-cost polluting firm reacts by simply producing in this period the output that equals the profit-maximizing level as a monopolist, $q_{1L} = q_{1L}^m$. In other words, the informational asymmetry imposes a signaling cost on the high-cost firm, i.e. to the one that pollutes a little. Hence, signaling reduces the expected production of period 1 with respect to the level it would be if the polluting firm behaved as a simple monopolist and, as a consequence, reduces the environmental damage. This is the so-called non-trivial separating equilibrium (in short, NTSE). In contrast with this, when the high-cost firm is very inefficient compared with the low-cost firm, it need not produce less than q_{1H}^m to separate itself from the low-cost firm, in which case the separating equilibrium that arises is a trivial separating equilibrium (henceforth TSE). Figure 1 depicts these two regions.

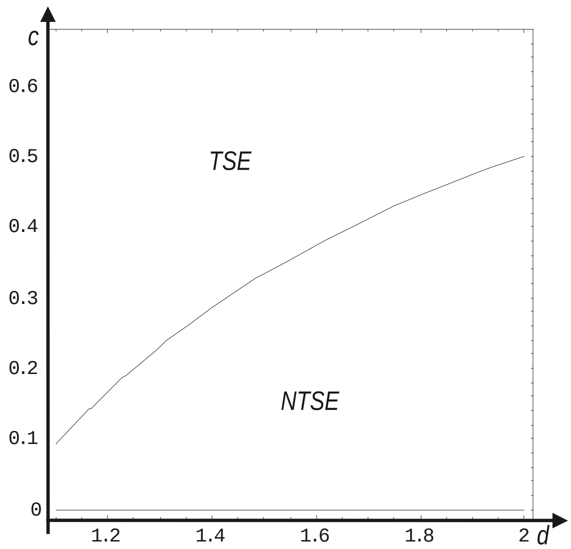

Fig. 1. Regions of Trivial and Non-Trivial Separating Equilibrium in the {*d*,*c*}-space (1<*d*<2)

Now, the policymaker takes this unique separating equilibrium as given and determines the environmental tax that maximizes its objective function over this period. Such an optimal environmental tax is the one from the following lemma, where superscript *S* denotes the separating equilibrium framework.

Lemma 3. *When* $d>1$ *and the polluter signals its cost through the output it produces at t*=1*, the optimal pollution tax for such a period is as follows:*

(i) $e_1^S=((d-1)/(d+1))\,(1-(1-\gamma)\sqrt{(d-1)\,[4+(d-1)c]\,c}\,/(d+1))$, *if parameters d and c satisfy the condition* $(d+1)c-\sqrt{(d-1)\,[4+(d-1)c]\,c}<0$*;*

(ii) $e_1^S=((d-1)/(d+1))[1-(1-\gamma)c]$, *otherwise.*

Proof. See the Appendix.

Behind this environmental tax is the consideration of the environmental damage and the two above-mentioned signaling effects: the direct-signaling effect and the strategic-signaling effect. Part (i) of the lemma refers to the optimal environmental tax in the region of parameters (*d,c*) where a NTSE prevails. In this case, the optimal tax increases as parameter *d* increases, because this leads the policymaker to value the environmental damage more and the polluting firm to have a higher incentive to be perceived as a high-cost firm. This implies in turn an increase in the signaling cost of the high-cost firm as the policymaker increases the amount of the tax. The size of the environmental tax also increases as the probability of the polluting firm being efficient increases, because this makes it less likely that a signaling cost exists. However, the optimal environmental tax decreases as parameter *c* increases, because the signaling cost decreases. In particular, when parameter *c* is high enough for a TSE to prevail, none of the cited effects hold, and hence the optimal tax will be lower than in the region where a NTSE exists.

Turning back to Lemma 2, the firm's output and pollutant emissions at *t*=1 are given by

$$q_{1H}^S=\begin{cases}\dfrac{1}{d+1}-\dfrac{(2-\gamma+d\gamma)\sqrt{(d-1)\,[4+(d-1)c]\,c}}{2(d+1)^2}, & \text{if } (d+1)c-\sqrt{(d-1)\,[4+(d-1)c]\,c}<0\\[2ex] \dfrac{1}{d+1}-\dfrac{(2-\gamma+d\gamma)c}{2(d+1)}, & \text{otherwise,}\end{cases} \tag{17}$$

if the firm is inefficient, and by

$$q_{1L}^{S}=\begin{cases}\dfrac{1}{d+1}+\dfrac{(d-1)(1-\gamma)\sqrt{(d-1)[4+(d-1)c]c}}{2(d+1)^{2}}, & \text{if } (d+1)c-\sqrt{(d-1)[4+(d-1)c]c}<0\\ \dfrac{1}{d+1}+\dfrac{(d-1)(1-\gamma)c}{2(d+1)}, & \text{otherwise,}\end{cases} \tag{18}$$

if it is efficient.

From (17) and (18) it immediately follows that under conditions of part (i) of Lemma 3 both the expected output level and the expected emissions in period 1 are lower than in the case in which the polluting firm behaved as a simple monopolist.

4. Pooling sequential equilibria

In a pooling sequential equilibrium, both the high-cost and the low-cost type of polluter decide to choose the same output level in period 1. So, no information can be inferred by the policymaker from the observation of the firm's decision and its updated belief of the polluter being low-cost equals its prior assessment. A pooling sequential equilibrium is, in this framework, a list of actions and beliefs $\{e_1(\tilde{c}),\, q_1(\tilde{c},e_1),\, \gamma(q_1),\, e_2(\tilde{c}),\, q_2(\tilde{c},e_2),\, e_2(\tilde{c})\}$ of the form

$$e_1(\tilde{c})=e_1, \text{ for all } \tilde{c}\in\{0,c\}, \tag{19}$$

$$q_1(\tilde{c},e_1)=q_1, \text{ for all } \tilde{c}\in\{0,c\}, \tag{20}$$

$$\gamma(q_1)=\gamma, \tag{21}$$

$$e_2(\tilde{c})=e_2, \text{ for all } \tilde{c}\in\{0,c\}, \tag{22}$$

and

$$q_2(\tilde{c},e_2)=\begin{cases}q_{2L}^{P}, \text{if } \tilde{c}=0\\ q_{2H}^{P}, \text{if } \tilde{c}=c,\end{cases} \tag{23}$$

where superscript *P* denotes a pooling equilibrium. As usual, such an equilibrium is determined by using the classical backwards induction argument.

Period 2

In this period, the system of updated beliefs of the policymaker is the one defined in (3). So, given the output produced by the polluter in period 1, which is the same than that defined in (10), the problem the regulator seeks to solve is

$$\max_{e_2}\ W_2 = \frac{1}{2}\left[\gamma\left(\frac{1-e_2}{2}\right)^2 + (1-\gamma)\left(\frac{1-c-e_2}{2}\right)^2\right] + \gamma\left(\frac{1-e_2}{2}\right)^2 + (1-\gamma)\left(\frac{1-c-e_2}{2}\right)^2$$
$$+e_2\left[\gamma\left(\frac{1-e_2}{2}\right) + (1-\gamma)\left(\frac{1-c-e_2}{2}\right)\right] - \frac{1}{2}d\left[\gamma\left(\frac{1-e_2}{2}\right)^2 + (1-\gamma)\left(\frac{1-c-e_2}{2}\right)^2\right], \quad (24)$$

and the optimal pollution tax, which follows straightforwardly from (24), is that of the following lemma.

Lemma 4. *When* $d > 1$ *and the firm's output of period* 1 *does not signal the firm's marginal cost, the optimal environmental tax chosen for period* 2 *is* $e_2^P = ((d-1)/(d+1))\,[1-(1-\gamma)c]$.

When choosing the optimal environmental tax in a non-signaling context, the policymaker takes into account the fact that the firm's expected output of period 2 is the one that simply corresponds to a monopoly. Thus, in the trade-off between underproduction due to the exercise of market power and environmental damage caused by production, it sets a positive environmental tax. As might have been expected, the amount of such an environmental tax increases as parameter *d* increases due to the increase in the regulator's ecological conscience. Likewise, an increase in parameter γ leads to an increase in the environmental tax because the firm's expected output, and, thus the firm's pollutant emissions, increase. Finally, the environmental tax decreases as parameter *c* increases since the expected level both of the production and the pollutant emitted decrease.

Substitution of this environmental tax into the outputs defined in (10) affords the output (and emissions) levels given by

$$q_{2H}^P = \frac{2(1-c)-\gamma(d-1)c}{2(d+1)}, \quad (25)$$

if the polluter is of high-cost type, and

$$q_{2L}^{P} = \frac{2+(1-\gamma)(d-1)c}{2(d+1)}, \tag{26}$$

if it is a low-cost firm.

Period 1

The polluter's output of period 1 forms part of a pooling sequential equilibrium whenever both types of the firm select the same output level in such a period. An output such as $q_1^P \in [q_{1H}^m, q_{1L}^m]$, with associated posterior beliefs $\gamma(q_1^P) = \gamma$ jointly with the out-of-equilibrium beliefs $\gamma(q_1) = 1$ if $q_1 < q_1^P$ and $\gamma(q_1) = 0$ if $q_1 > q_1^P$ should be proposed as a candidate to a pooling equilibrium. Nevertheless, none of them survive once the equilibrium-dominated outputs at establishing out-of-equilibrium beliefs are eliminated. To see why, it suffices to consider that the output level q_1^P forms part of a pooling only because $\gamma(q_{1H}^m) = 1.$ The low-cost polluter, however, finds that the output q_{1H}^m is dominated by the output q_1^P since the profit function $\Pi_{1L}(q_1)$ is strictly concave in the output and reaches a maximum at level $q_1 = q_{1L}^m$ satisfying $q_{1L}^m > q_1^P > q_{1H}^m$. So, the only possible posterior belief the policymaker may adopt after observing output level q_{1H}^m is $\gamma(q_{1H}^m) = 0$ rather than $\gamma(q_{1H}^m) = 1.$ The pooling equilibrium proposed above is then broken.

5. Comparing the signaling and non-signaling equilibria

If the output produced by the polluter in period 1 is not a signal of its cost, then the output defining an equilibrium would correspond, in both production periods, to the output level derived in (25) and (26) above. Similarly, the optimal environmental tax would be, in both periods, the one derived in Lemma 4 above. As a consequence, any difference between environmental taxes e_1^S and e_2^P, and outputs q_{1H}^S and q_{2H}^P, and q_{1L}^S and q_{2L}^P must be entirely attributed to the role of the output produced in period 1 as a signal of the firm's costs. Regarding the environmental tax, the result is recorded in the following proposition.

Proposition 1. *When $d > 1$ the optimal environmental tax rate settled by the policymaker in a dynamic setting is as follows:*[15]

(i) $e_1^S < e_2^P$, *if parameters d and c satisfy the condition* $(d+1)c - \sqrt{(d-1)[4+(d-1)c]\,c} < 0$*;*

(ii) $e_1^S = e_2^P$, *if parameters d and c satisfy* $(d+1)c - \sqrt{(d-1)[4+(d-1)c]\,c} \geq 0$.

Proof. Straightforward from Lemmas 3 and 4.

The intuition behind this proposition is quite simple. In the NTSE, the signaling cost that the high-cost polluting firm (i.e., the one that pollutes a little) suffers is understood by the policymaker, which reduces the environmental tax rate with respect to the case where signaling is absent in order to decrease such a cost. In other words, if the environmental tax were increased in the signaling context, the high-cost firm would find it more difficult to be perceived as such. It would then be forced to reduce even more its period 1 output to separate itself from the low-cost firm, by which output in period 1 would be depressed even further from the expected level. This would be beneficial for the environmental damage, but harmful for the consumer surplus, the firm's profit and the taxes accrued by the policymaker. Given that the sum of consumer surplus, the firm's profits and the policymaker's income outweigh the environmental damage, the policymaker significantly decreases the amount of environmental tax in the signaling context.

Part (ii) of the proposition states, in turn, that when the signaling process is costless, any difference between the amount of e_1^S and that of e_2^P disappears. Figure 2 depicts the regions of parameters in the $\{d,c\}$-space in which a TSE and a NTSE exist.

[15] It can be shown that the optimal environmental tax rate is always placed below the marginal environmental cost irrespective of the signaling or non-signaling context. This result, which parallels Barnett (1980), Kennedy (1994) and Bárcena and Garzón (2002), among others, is due to the fact that the production level is already diminished by the exercise of market power, which leads the policymaker to choose an environmental tax to not reduce the output further.

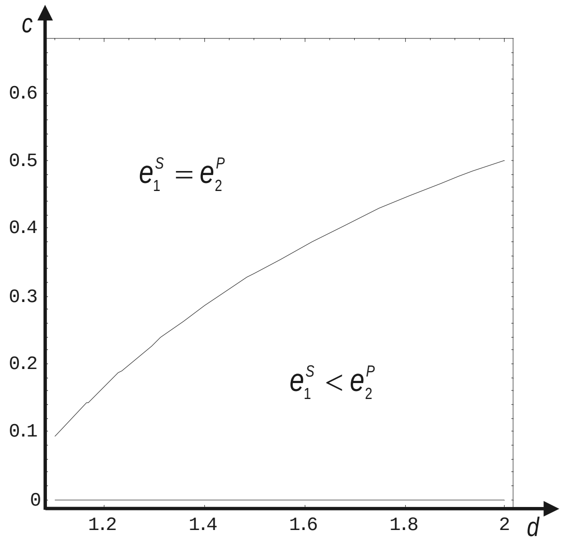

Fig. 2. Optimal environmental tax in the {*d*,*c*}-space when the firm's output signals its costs and viceversa

The under-taxation existing in the signaling context increases, other things being equal, as parameter *d* increases. This is because any increase in parameter *d* reduces the size of the signaling effects. Likewise, the under-taxation decreases as parameters *c* increases because an increase in the gap of technological efficiency reduces the signaling cost of the high-cost firm. Finally, the under-taxation decreases as parameter γ increases, since the expected output and pollution emitted approximate those existing in the non-signaling context. All of this can be illustrated in the following numerical example:

$\tilde{c}$	γ	d	e_1^S	e_2^P	e_{2H}^S	e_{2L}^S
{0, .25}	.10	1.1	.037	.037	.036	.048
		1.5	.148	.155	.150	.200
		2.0	.230	.258	.250	.333
	.50	1.1	.042	.042	.036	.048
		1.5	.171	.175	.150	.200
		2.0	.276	.292	.250	.333
	.99	1.1	.047	.047	.036	.048
		1.5	.199	.199	.150	.200
		2.0	.332	.332	.250	.333
{0, .5}	.10	1.1	.026	.026	.024	.048
		1.5	.126	.110	.100	.200
		2.0	.183	.183	.167	.333
	.50	1.1	.036	.036	.024	.048
		1.5	.159	.150	.100	.200
		2.0	.250	.250	.167	.333
	.99	1.1	.047	.047	.024	.048
		1.5	.199	.199	.100	.200
		2.0	.332	.332	.167	.333

Table 1. A numerical example for the case in which *d*>1.

Next, we shall examine how the production level, environmental damage, and social welfare are affected by the fact that the polluting firm signals its costs.

Corollary 1. (i) *Compared to the non-signaling context, the following holds in the NTSE of the signaling game:*

(i.1) $q_1^S < q_2^P$;

(i.2) $CS_1^S < CS_2^P$;

(i.3) $\Pi_1^S < \Pi_2^P$;

(i.4) $W_1^S < W_2^P$.

(ii) *If a TSE prevails, no difference occurs between the signaling and non-signaling contexts.*

Proof. See the Appendix.

Despite the fact that under signaling the policymaker levies a lower environmental tax than in the non-signaling context, the firm's expected output of period 1 in the former context is not greater than in the latter. As a consequence, the expected pollutant emitted is not higher in the signaling context than in the non-signaling one. Similar reasoning applies to the expected consumer surplus, the firm's profit and the level of social welfare.

6. The case in which the valuation of the environmental quality is low

When the policymaker's valuation of the environmental quality is sufficiently low, the environmental tax for period 2 becomes negative (a subsidy per unit of pollutant emitted).[16] In this case, the high-cost firm is interested in misrepresenting itself as a low-cost firm as a means to increase the amount of the subsidy received in period 2 and, thus, its profits in the second period (see Lemma 1).

Under these circumstances the conditions needed for a separating equilibrium to exist are given by

$$q_{1H} = q_{1H}^m, \tag{27}$$

[16] See expression (12) on p. 12.

$$\Pi_{1L}^{m} - \Pi_{1L}(q_{1L}) \leq \Pi_{2L}(q_{2L} \mid \gamma(q_{1L}) = 1) - \Pi_{2L}(q_{2L} \mid \gamma(q_{1L}^{m}) = 0)\,, \tag{28}$$

and

$$\Pi_{1H}^{m} - \Pi_{1H}(q_{1H}) \geq \Pi_{2H}(q_{2H} \mid \gamma(q_{1L}) = 1) - \Pi_{2H}(q_{2H} \mid \gamma(q_{1H}^{m}) = 0)\,. \tag{29}$$

Condition (27) indicates that the best a high-cost polluting firm can do in period 1 is to produce the profit-maximizing output as a simple monopolist. In turn, inequality (28) is the incentive compatibility condition for a low-cost monopolist and states that it would prefer to produce output q_{1L} in period 1 (an output that may or may not differ from the profit-maximizing output as a simple monopolist, q_{1L}^{m}) and convince the policymaker it is a low-cost firm, rather than produce the output that maximizes its profit in period 1 as a low-cost monopolist and then be perceived as a high-cost firm. Finally, what condition (29) indicates is that the high-cost firm would prefer to produce output q_{1H}^{m} in period 1 and be perceived as a high-cost firm, rather than be perceived as a low-cost firm through being obliged to produce output q_{1L} in period 1. The resolution of these conditions enables us to obtain the period 1 outputs that form part of a separating sequential equilibrium. Such outputs are summarized in the following lemma.

Lemma 5. *When* $d < 1$*, the polluter's output in period* 1 *that forms part of a separating sequential equilibrium is as follows:*

(i) $q_{1L} = (1-c-e_1)/2 + \sqrt{(1-d)[4-(3+d)c]\,c}\,/2(1+d)$ *when the firm is of low-cost type, and* $q_{1H} = (1-c-e_1)/2 \equiv q_{1H}^{m}$ *when it is of high-cost, if parameters d and c satisfy the condition* $(d+1)c - \sqrt{(1-d)\,[4-(3+d)c]\,c} < 0$*;*

(ii) $q_{1L} = (1-e_1)/2 \equiv q_{1L}^{m}$ *when the firm is of low-cost type and* $q_{1H} = (1-c-e_1)/2 \equiv q_{1H}^{m}$ *when it is a high-cost firm, if parameters d and c satisfy* $(d+1)c - \sqrt{(1-d)\,[4-(3+d)c]\,c} \geq 0$*.*

Proof. See the Appendix.

In this case, the incentive to disclose a low-cost leads the efficient firm to separate itself from the inefficient one by increasing its production in period 1 above the monopolist's level, whenever the efficiency gap is not very high (the so-called NTSE referred to in part (i) of the lemma) or by simply producing the profit-maximizing level of a monopolist if the technological efficiency gap is high enough (the TSE contained in part (ii)).

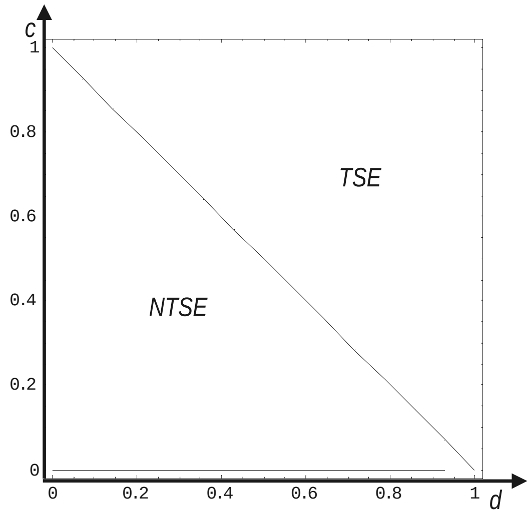

Fig. 3. Regions of Trivial and Non-Trivial Separating Equilibrium when $d<1$

Next, we are able to determine the optimal environmental tax in the signaling context.

Lemma 6. *When* $d<1$ *and the polluter signals its cost through the output it produces at t=1, the optimal pollution tax for such a period is negative (a subsidy per unit of pollutant emitted) and given by*

(i) $e_1^S=((1-d)/(1+d))\left(c-1-\gamma\sqrt{(1-d)[4-(3+d)c]c}/(1+d)\right)$, *if parameters d and c satisfy the condition* $(1+d)c-\sqrt{(1-d)[4-(3+d)c]c}<0$*;*

(ii) $e_1^S=((1-d)/(1+d))[c-1-\gamma c]$, *otherwise.*

Proof. See the Appendix.

As mentioned before, the amount of the subsidy in this context depends on the importance of the underproduction in the light of the environmental damage, the direct-signaling effect, which leads the low-cost firm to set its output level above the short-term optimum in period 1, and the strategic-signaling effect, which provides the incentive for the high-cost firm to mimic a low-cost firm. The size of the subsidy decreases as parameter *d* increases since the valuation of the underproduction also decreases and the incentive of the firm to be understood as a low-cost firm is reduced. The subsidy also decreases as the efficiency gap of the technology increases because it both reduces output and the incentive of the polluter to be taken by the policymaker as a low-cost firm. Finally, the subsidy increases with the probability of a low-cost outcome since this increases the output produced and the amount of the two signaling effects.

Taking into account both Lemma 6 above and the subsidy per unit of pollution emitted set in a non-signaling context, $e_2^P = [(1-d)(c-1-\gamma c)]/(1+d)$, we can establish the following proposition.

Proposition 2. *The optimal environmental tax settled by the policymaker in a dynamic context when* $d<1$ *is negative (a subsidy per unit of pollutant emitted) and evolves as follows:*

(i) $|e_1^S| > |e_2^P|$, *if parameters d and c satisfy the condition* $(d+1)c - \sqrt{(1-d)[4-(3+d)c]c} < 0$*;*

(ii) $|e_1^S| = |e_2^P|$, *if parameters d and c satisfy* $(d+1)c - \sqrt{(1-d)[4-(3+d)c]c} \geq 0$.

Part (i) refers to the case in which a NTSE exists. In this case, the signaling cost of the efficient firm that produces more than it would otherwise, in order to convey information to the policymaker, has the effect of increasing the amount of the subsidy as compared to the non-signaling context (direct-signaling effect). The policymaker also increases the size of the subsidy in the signaling case with the aim of increasing the incentive of the high-cost firm to be perceived as a low-cost firm (strategic-signaling effect). In turn, part (ii) summarizes the case in which a TSE exists and thus both signaling effects disappear. This result is depicted in Figure 4

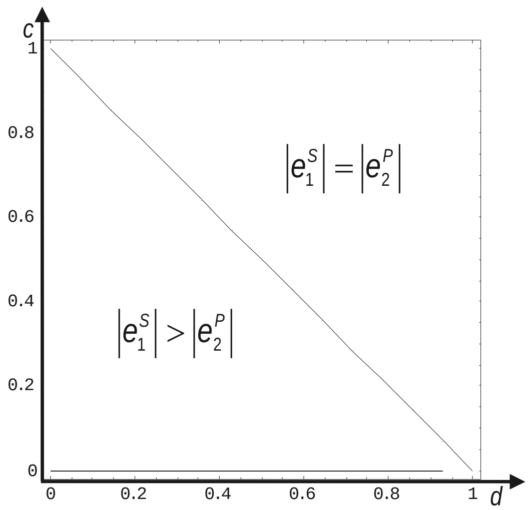

Fig. 4. Optimal environmental subsidy in the {*d*,*c*}-space when the firm's output signals its costs and viceversa

and illustrated in the following numerical example

$\tilde{c}$	γ	d	e_1^S	e_2^P	e_{2L}^S	e_{2H}^S
{0, .25}	.10	.01	-0.822	-0.760	-0.980	-0.735
		.50	-0.264	-0.258	-0.333	-0.250
		.99	-0.004	-0.004	-0.005	-0.004
	.50	.01	-1.170	-0.858	-0.980	-0.735
		.50	-0.319	-0.292	-0.333	-0.250
		.99	-0.004	-0.004	-0.005	-0.004
	.99	.01	-1.596	-0.978	-0.980	-0.735
		.50	-0.387	-0.332	-0.333	-0.250
		.99	-0.005	-0.005	-0.005	-0.004
{0, .80}	.10	.01	-0.305	-0.274	-0.980	-0.196
		.50	-0.093	-0.093	-0.333	-0.067
		.99	-0.001	-0.001	-0.005	-0.001
	.50	.01	-0.741	-0.588	-0.980	-0.196
		.50	-0.003	-0.003	-0.333	-0.067
		.99	-0.003	-0.003	-0.005	-0.001
	.99	.01	-1.275	-0.972	-0.980	-0.196
		.50	-0.331	-0.331	-0.333	-0.067
		.99	-0.005	-0.005	-0.005	-0.001

Table 2. A numerical simulation when d<1.

Corollary 2. (i) *When* $d<1$ *and compared to the non-signaling context, the following holds in the NTSE*:

(i.1) $q_1^S > q_2^P$;

(i.2) $CS_1^S > CS_2^P$;

(i.3) $\Pi_1^S > \Pi_2^P$;

(i.4) $W_1^S > W_2^P$.

(ii) *If a TSE prevails, no difference occurs between the signaling and non-signaling contexts.*

Proof. See the Appendix.

Results of part (i) are due to the fact that under signaling the policymaker levies a lower environmental tax than in the non-signaling context. Thus, the firm's expected output of period 1 in the former context is greater than in the latter and, as a consequence, the expected pollutant emitted is also higher in the signaling context than in the non-signaling one, as well as the expected consumer surplus, the firm's profit and the level of social welfare. In the case in which the signaling is costless, the subsidy per unit of pollutant emitted is the same in both contexts and, thus, the industry's performance is not affected.

7. Concluding remarks

The analysis of pollution taxes and charges is relevant not only from the perspective of theories related to the environment, but also for political discussion. Policymakers need to understand how such instruments work when considering whether or not they should be introduced in the future and to what extent. The main purpose of this paper has been to examine the optimal pollution tax/subsidy policy in a polluting monopolistic industry in which the emitter's costs are unobservable for the policymaker. In this case there is vertical asymmetric information, and the firm may or may not signal its costs through its output choices. An extensive search of the literature has not revealed any research into models of optimal environmental taxation that have considered this problem. The assumption that firms have private information and can transmit it by their behavior complicates the issue, since the policymaker must consider the signaling effects when the tax to be paid by the polluting firm is chosen. However, the assumption that firms have private information that the policymaker can only infer permits a closer approximation to real-life situations than the assumptions commonly made.

In this context, a number of scenarios are examined depending upon the level of costs of the firm and the value attached to environmental degradation by the policymaker. The results shown above indicate that whenever the policymaker places a high value on environmental damage, the environmental tax in the case in which the monopolist signals its costs of production through the output it produces in the first period is not higher than when the firm does not signal such costs. The incentive of the monopolistic polluter to be perceived by the policymaker as a high-cost firm leads it to under-produce in the first period when the gap of technological efficiency is low enough (or simply to produce the non-signaling level in the first period when the gap is sufficiently high). Thus, in order to make it easier for the high-cost monopolist to be perceived as such, the policymaker reduces the environmental tax rate in the signaling framework with respect to the non-signaling one. In turn, when the policymaker's valuation of the environmental damage is so low that consumption outweighs environmental damage, the pollutant tax becomes negative (a subsidy per unit of pollutant emitted) in both contexts, and its size in the signaling case is greater than or equal to that in the non-signaling context.

This approach opens up a number of avenues for further research. For example, the model could be used to analyze the choice of the environmental tax/subsidy policy when multiple polluting firms co-exist and there is asymmetric information not only between each firm and the policymaker, as in the current paper, but also among polluting firms. This is left for future research.

Appendix

Proof of Lemma 1. If the polluting firm is of low-cost type but it has signaled itself as a high-cost firm and, consequently, it must pay in period 2 the environmental tax designed for a high-cost firm, then its maximized profit in period 2 is given by

$$\Pi_{2L}(q_{2L}|\gamma(\cdot)=0)=\left(\frac{2+(d-1)c}{2(d+1)}\right)^2$$

$$=\left(q_{2L}+\frac{(d-1)c}{2(d+1)}\right)^2. \tag{A1}$$

Similarly, if the emitter is of high-cost type, but the policymaker believes that it is of low-cost firm, then its maximized profit in period 2 is

$$\Pi_{2H}(q_{2H}|\gamma(\cdot)=1)=\left(\frac{2-(d+1)c}{2(d+1)}\right)^2$$

$$=\left(q_{2H}-\frac{(d-1)c}{2(d+1)}\right)^2. \tag{A2}$$

From (A1) and (A2), the result claimed in the lemma follows. ■

Proof of Lemma 2. Particularizing the incentive compatibility conditions defined in (14)-(16), we have, respectively,

$$q_{1L}=\frac{1-e_1}{2}\equiv q_{1L}^m, \tag{A3}$$

$$(1-c-e_1-q_{1H})q_{1H}+\left(\frac{1-c}{d+1}\right)^2\geq\left(\frac{1-c-e_1}{2}\right)^2+\left(\frac{2-(d+1)c}{2(d+1)}\right)^2, \tag{A4}$$

and

$$\left(\frac{1-e_1}{2}\right)^2+\left(\frac{1}{d+1}\right)^2\geq(1-e_1-q_{1H})q_{1H}+\left(\frac{2+(d-1)c}{2(d+1)}\right)^2. \qquad (A5)$$

By taking condition (A4) as equality, the resulting second-degree equation has

$$q_{1H}=\frac{1-c-e_1}{2}\pm\frac{\sqrt{(d-1)[4-(d+3)c]\,c}}{2(d+1)}, \qquad (A6)$$

as roots, while the second-degree equation formed by taking condition (A5) as equality has the following roots

$$q_{1H}=\frac{1-e_1}{2}\pm\frac{\sqrt{(d-1)[4+(d-1)c]\,c}}{2(d+1)}. \qquad (A7)$$

Denote by r^- the lowest root of the two obtained in (A6) and by s^- the lowest root of the two defined in (A7). Given that $r^-<s^-$, the continuum of separating equilibria is formed by the interval of outputs given by $q_{1H}\in[r^-,s^-]$. On the other hand, any period 1 output strictly less than s^- is a dominated-output for the high-cost polluting firm and thereby the separating equilibrium of minimum cost involves the output $q_{1H}=s^-$. It remains now to check when the equilibrium is a NTSE or a TSE. It is easy to check that $q_{1H}^m\equiv(1-c-e)/2<s^-$, when parameters *d* and *c* verify $(d+1)c-\sqrt{(d-1)[4+(d-1)c]\,c}<0$. In such a case, period 1 outputs of the polluter that form part of the NTSE are $q_{1H}=s^-$ and $q_{1L}=q_{1L}^m$, which refers to part (i) of the lemma. Similarly, $q_{1H}^m\geq s^-$ when parameters *d* and *c* verify the condition $(d+1)c-\sqrt{(d-1)[4+(d-1)c]\,c}\geq 0$ in which case the productions of the emitter as a simple monopolist, i.e. q_{1H}^m if it is a high-cost firm and q_{1L}^m if it is a low-cost firm, form part of a separating sequential equilibrium. This is the so-called TSE of part (ii) of the lemma. ∎

Proof of Lemma 3. Given the objective function of the policymaker defined in (4), the problem

$$\max_{e_1} W_1=(1-d)[\gamma(q_{1L})^2+(1-\gamma)(q_{1H})^2]/2+\gamma(q_{1L})^2+(1-\gamma)(q_{1H})^2+\gamma e_1q_{1L}+(1-\gamma)e_1q_{1H} \qquad (A8)$$

where q_{1L} and q_{1H} are the equilibrium outputs defined in Lemma 2, has

$$0=\frac{\partial W_1}{\partial e_1}=(3-d)\left(\gamma q_{1L}\frac{\partial q_{1L}}{\partial e_1}+(1-\gamma)q_{1H}\frac{\partial q_{1H}}{\partial e_1}\right)+\gamma\left(q_{1L}+e_1\frac{\partial q_{1L}}{\partial e_1}\right)+(1-\gamma)\left(q_{1H}+e_1\frac{\partial q_{1H}}{\partial e_1}\right) \quad \text{(A9)}$$

as first-order condition. Finally, solving (A.9) yields the result claimed in the lemma. ■

Proof of Lemma 4. Solving the first-order condition of problem (24),

$$0=\frac{\partial W_2}{\partial e_2}=-\gamma(1-e_2)-(1-\gamma)(1-c-e_2)-\gamma(2-2e_2)-(1-\gamma)(2-2c-2e_2)+\gamma(2-2e_2)-2\gamma e_2$$
$$+(1-\gamma)(2-2c-2e_2)-2(1-\gamma)e_2+d\gamma(1-e_2)+d(1-\gamma)(1-c-e_2)\,, \quad \text{(A10)}$$

the result holds.

Proof of Corollary 1

(i.1) From (17)-(18) and (25)-(26), we have

$$q_1^S-q_2^P=\gamma q_{1L}^S+(1-\gamma)q_{1H}^S-(\gamma q_{2L}^P+(1-\gamma)q_{2H}^P)=\frac{(1-\gamma)(c(d+1)\sqrt{(d-1)c(4-(d+3)c)})}{(d+1)^2} \quad \text{(A11)}$$

which is negative by virtue of Proposition 1.

(i.2) By comparing $CS_1^S=[\gamma(q_{1L}^S)^2+(1-\gamma)(q_{1H}^S)^2]/2$ and $CS_2^P=[\gamma(q_{2L}^P)^2+(1-\gamma)(q_{2H}^P)^2]/2$.

(i.3) The firm's profit in equilibrium is $\Pi_1^S=\gamma(q_{1L}^S)^2+(1-\gamma)(q_{1H}^S)^2$ in the signaling game, and $\Pi_2^S=\gamma(q_{2L}^P)^2+(1-\gamma)(q_{2H}^P)^2$ in the non-signaling game. Comparing them, the result yields.

(i.4) In the signaling game, the level of social welfare in equilibrium is given by $W_1^S=((3-d)/2)\,[\gamma(q_{1L}^S)^2+(1-\gamma)(q_{1H}^S)^2]+e_1^S[\gamma q_{1L}^S+(1-\gamma)q_{1H}^S]$, whereas in the non-signaling game is $W_2^P=((3-d)/2)\,[\gamma(q_{2L}^P)^2+(1-\gamma)(q_{2H}^P)^2]+e_2^P[\gamma q_{2L}^P+(1-\gamma)q_{2H}^P]$. Tedious manipulation yields $W_1^S<W_2^P$, whenever $(d+1)c-\sqrt{(d-1)[4+(d-1)c]\,c}<0$.

(ii) Straightforward. ■

Proof of Lemma 5. Once the incentive compatibility conditions (28)-(29) are particularized, they become, respectively,

$$\left(\frac{1-e_1}{2}\right)^2-(1-e_1-q_{1L})q_{1L}\leq\left(\frac{1}{d+1}\right)^2-\left(\frac{2-(1-d)c}{2(d+1)}\right)^2 \tag{A12}$$

for the emitter of low-cost, and

$$\left(\frac{1-c-e_1}{2}\right)^2-(1-c-e_1-q_{1L})q_{1L}\geq\left(\frac{2-(d-1)c}{2(d+1)}\right)^2-\left(\frac{1-c}{d+1}\right)^2 \tag{A13}$$

for the high-cost firm. When condition (A12) is solved as equality, it yields the roots

$$q_{1L}=\frac{1-e_1}{2}\pm\frac{\sqrt{c(1-d)[4-c(1-d)]}}{2(d+1)} \tag{A14}$$

and, similarly, condition (A13) affords

$$q_{1L}=\frac{1-c-e_1}{2}\pm\frac{\sqrt{c(1-d)[4-c(3+d)]}}{2(d+1)}\,. \tag{A15}$$

Denote by v^+ the highest root of the two defined in (A14) and by u^+ the highest root of the two defined in (A15). Given that $v^+>u^+$, the interval $q_{1L}\in[u^+,v^+]$ defines the continuum of separating equilibria. Among them, the separating equilibrium of minimum cost is $q_{1L}=u^+$. Finally, it holds that $u^+>(1-e_1)/2\equiv q_{1L}^m$ when parameters d and c satisfy $\sqrt{c(1-d)[4-c(3+d)]}>c(d+1)$, in which case a TSE emerges, while $u^+\leq(1-e_1)/2\equiv q_{1L}^m$ when $\sqrt{c(1-d)[4-c(3+d)]}\leq c(d+1)$, in which case there is a NTSE. ■

Proof of Lemma 6. (i) By solving condition (A9) once evaluated in $(q_{1L},q_{1H})=\left(\frac{1-e_1}{2}+\frac{\sqrt{c(1-d)[4-c(1-d)]}}{2(d+1)},\frac{1-c-e_1}{2}\right)$ when $\sqrt{c(1-d)[4-c(3+d)]}-c(d+1)>0$

(ii) By solving condition (A9) once it has been evaluated in $(q_{1L}, q_{1H}) = \left((1-e_1)/2, (1-c-e_1)/2\right)$.

■

Proof of Corollary 2.

(i.1) From Lemma 5, both firm's production and pollutant emissions in period 1 are given by

$$q_{1L}^{S} = \begin{cases} \dfrac{1-c}{1+d} + \dfrac{(1+d+\gamma-d\gamma)\sqrt{c(1-d)\,[4-c(3+d)]}}{2(1+d)^2}, & \text{if } (1+d)c - \sqrt{c(1-d)\,[4-c(3+d)]} < 0 \\ \dfrac{1}{1+d} - \dfrac{(1-d)(1-\gamma)c}{2(1+d)}, & \text{otherwise,} \end{cases} \tag{A16}$$

if the emitter is a low-cost firm, and by

$$q_{1H}^{S} = \begin{cases} \dfrac{1-c}{1+d} + \dfrac{(1-d)\gamma\sqrt{c(1-d)[4-(3+d)c]}}{2(1+d)^2}, & \text{if } (1+d)c - \sqrt{(1-d)[4-(3+d)c]c} < 0 \\ \dfrac{1-c}{1+d} + \dfrac{(1-d)\gamma c}{2(1+d)}, & \text{otherwise,} \end{cases} \tag{A17}$$

if it is a high-cost firm. In turn, in the non-signaling context, output levels (and pollutant emissions) are

$$q_{2L}^{P} = \frac{2-(1-d)(1-\gamma)c}{2(1+d)} \tag{A18}$$

for the low-cost firm, and

$$q_{2H}^{P} = \frac{2-(2-\gamma+d\gamma)c}{2(1+d)} \tag{A19}$$

for the high-cost. Then, from (A16)-(A19) we obtain

$$q_1^S - q_2^P = \frac{\sqrt{c(1-d)[4-c(3+d)]}-(1+d)c}{(1+d)^2} \qquad \text{(A20)}$$

and $q_1^S > q_2^P$ by virtue of Proposition 2.

(i.2) and (i.3) Straightforward.

(i.4) Evaluating the level of social welfare in both regimes, we have

$$W_1^S - W_2^P = \frac{\gamma(1+d)\begin{bmatrix}2(1-c)\sqrt{c(1-d)[4-c(3+d)]}+c(1-d)(1-c-d)\gamma \\ +c[1-(6-d)d-c(1-3d)]\end{bmatrix}}{2(1+d)^3} \qquad \text{(A21)}$$

which is positive in the region where $\sqrt{c(1-d)[4-c(3+d)]} \leq c(d+1)$.

(ii) Immediate. ■

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