DOCUMENTO DE TRABAJO. E2005 / 25

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Depósito Legal: SE-6374-05

Documento de Trabajo
Serie Economía E2005/25

Double informational asymmetry, signaling, and environmental taxes*

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RESUMEN
Este artículo analiza el efecto que en los impuestos ambientales produce la señalización de las empresas contaminantes cuando cada una de ellas tiene información privada sobre su coste de producción, mientras que cualquier otro agente (empresas rivales y regulador) sólo tienen una percepción subjetiva de dicho coste. En consecuencia, existe información asimétrica tanto horizontal como verticalmente, y cada empresa puede manipular estratégicamente la percepción de las empresas rivales y del regulador. Mostramos que si el parámetro de conciencia ecológica del regulador es suficientemente elevado, las empresas desean señalizarse como empresas con coste de producción bajo, para lo cual producirán un nivel elevado de output y emitirán un volumen elevado de contaminación. En este caso, los impuestos ambientales óptimos son mayores que en ausencia de señalización al objeto de que las empresas de bajo coste, en su intento por separarse de las empresas de alto coste (incrementando para ello su nivel de producción y, por tanto, de contaminación), reduzcan las distorsiones en su nivel de output y también de emisiones. Por el contrario, si el regulador valora el medioambiente menos que el consumo, los impuestos ambientales se vuelven negativos (un subsidio por unidad de contaminación emitida), pero cada empresa sigue teniendo incentivo a señalizarse ante la empresa rival y el regulador como una empresa de coste bajo. En este caso, si la cantidad producida por cada empresa señaliza sus costes de producción, es óptimo fijar un subsidio mayor que en el contexto de referencia de no-señalización.

Palabras clave: Empresas contaminantes, información asimétrica horizontal y vertical, señalización y no-señalización, impuestos ambientales

ABSTRACT
This paper examines the effect of signaling on environmental taxation when each polluter privately knows whether its production cost is low or high, whereas third parties (i.e. the rival firms and the regulator) have only a subjective perception on such a cost. Consequently, there is both horizontal and vertical asymmetric information, and each polluting firm can strategically manipulate both the competitor and the policymaker's prior cost perceptions. We show that if the policymaker's ecological conscience is sufficiently high, polluters wish to be perceived as low-cost firms and, to this end, they will produce a high output level and they will emit a high emissions level. Therefore, optimal pollution taxes are higher than would be the case if firms' costs were not

signaled in such a manner as to force low-cost polluters, in an attempt to distinguish themselves from high-cost polluters (by increasing their output level and their emissions level), to reduce the distortions in their production and also in their emissions levels. By contrast, if the policymaker values environmental quality less than consumption, environmental taxes become negative (a subsidy per unit of pollutant emitted), but each polluting firm continues to attempt to convince the other players (the rival firm and the regulator) that it is a low-cost supplier. In this case, if the quantity produced by each polluter signals its costs, over-subsiding holds as compared to the benchmark case of non-signaling.

Keywords: Polluting firms, horizontal and vertical asymmetric information, signaling and non-signaling, environmental taxes
JEL classification: D82, L13, Q28

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* I wish to thank Lluís Bru, Fernando del Río and an anonymous referee for their useful comments and suggestions that greatly improved this paper. Of course, the usual disclaimer applies. Financial support from the Xunta de Galicia (Grant PGIDIT03CSO20101PR) is also gratefully acknowledged.
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1. Introduction

Economic activity involves not only goods, but also causes negative external effects that producers and consumers often are unable to internalize by themselves. Therefore, environmental protection has became a priority and a challenge for many governments. Among the market-oriented instruments to protecting the natural environment from the increasing cost of human behavior, the pollution-taxes approach is possibly one of the most widely used methods. According to the OECD (OECD, 2001) and EU data (EC, 1998; EEA, 1996, 2000), the four countries which are particularly diligent in applying pollution taxes (Canada, Denmark, Finland and Norway) have 95 environmental taxes, a number that increases to 176, if we extend the list to the top ten countries in terms of diligence (the above four plus Austria, the Netherlands, UK, Sweden, Switzerland and Japan).

The issue of imposing pollution taxes to control emissions is the target of a long-standing debate in the environmental economics literature, dating back to Arthur C. Pigou's (1920) well-known treatment of the subject. In theory, pollution taxes have many advantages when compared to command-and-control policies,[1] and their analysis is of concern not only because of its practical importance, but also because of its topicality and socio-political effects. Indeed, in recent times, there has been a surge in the analysis of environmental taxation, usually under the assumption that emissions per unit of output are constant.[2] In addition, increased importance is being accorded to the effect of uncertainty and the link between uncertainty and responsive behavioral changes to pollution taxes. A salient example is Leung (1992) who examines a situation in which both polluter and victim have full information about each other's preferences and technology, but the regulator is faced with a lack of information. In this context, a policy of taxing both injurer and victim leads to an economically efficient pollution level, because it helps the uninformed regulator in the case of a sequential game. The polluter is taxed so as to redistribute revenues between polluter and victim, and the victim is taxed to prevent him from exaggerating the damage claim. Other writers discuss different possibilities in the context of simultaneous or sequential games. For instance, Barigozzi and Villeneuve (2004) focus on

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[1] They allow least-cost abatement, are generally more dynamic, implement the Polluter Pay Principle, raise revenue for the government, provide incentives for producers and consumers alike, and there is substantial historical experience of their application. See Poyago-Theotoky (2003).

[2] See Schöb (2003) for a survey.

the signaling effect of a tax when the agents are less informed than the policymaker about the effect of their consumption, and they show that optimal taxes cannot be implemented under asymmetrical information.

The purpose of this paper is to examine whether disclosure of polluting firms' costs affects the magnitude of environmental taxes imposed by a policymaker as compared to the case in which they are not disclosed. Firms which pollute in varying amounts according to their particular cost characteristics that affect their output and emissions levels will predictably attempt to alter the amounts of their production levels and, accordingly, their emissions as perceived by other players (including the policymaker) by misrepresenting their efficiency level. In other words, firms will certainly intent to influence the extent of pollution levels that other players believe they are emitting, and do not necessarily alter their actual levels. This represents a substantial policy-making problem when setting appropriate taxes, since the policymaker faces a conflict between expected social welfare and informational objectives that may significantly affect the magnitude of pollution taxes. More precisely, there seems to be a reciprocal influence between polluting firms and the government.

In the model considered, two firms produce a homogenous good through two production periods and a policymaker is concerned about the environment, therefore using per-unit taxes to maximize expected total welfare. Each polluting firm has confidential information about its own marginal cost of production (specifically, whether it is low or high). Third parties, on the other hand, have only some prior belief or perception in this respect. In particular, the policymaker who imposes, in each production period, an optimal environmental tax on each polluting firm for two possible cost situations (low-cost or high-cost), faces an adverse selection problem. In order to pay a lower pollution tax than if it had a high-cost condition, each firm always has an incentive to persuade both the policymaker and the rival firm that its cost is low. Given this adverse selection problem, the current paper examines whether optimal pollution taxes are or are not sensitive to the assumed information structure and to the flow of such information.

Our environmental taxes game has the following time frame. In period 1, before becoming aware of the type of polluters in the market, the policymaker announces and commits to a per-unit environmental tax for this period, and the polluting firms each produce an output level which generates a given emissions level. What happens in period 2 is examined in the light of two different scenarios. One is the benchmark situation of non-signaling, in which the period 1 output of each polluter is neither observed by the policymaker nor by the rival firm. Hence, the policymaker announces and commits to environmental taxes for period 2 without knowing the polluters' costs, whereby the environmental taxes chosen in the second period equal those chosen for the first one and they constitute a form of long-term tax policy.

The alternative scenario considered is the so-called signaling regime, in which the quantity produced by each firm in the first period is observed provides both by the rival firm and the policymaker and provides them with full information about its production cost and, consequently, about its emissions level. Once they update their probability assessment that a certain polluter has a given cost characteristic, the policymaker announces and commits, at the beginning of the second period, to a period 2 per-unit output environmental tax. Finally, each of the polluting firms selects the output level for period 2. In this case, the environmental taxes are selected period by period as information is obtained, a policy that may be regarded as a form of short-run tax policy.

We show that if the parameter measuring the ecological conscience of the policymaker is sufficiently high, optimal taxes imposed on polluters in period 2 are greater, the higher their revealed costs. Hence, each polluter has a vertical incentive to convince the policymaker that it is a low-cost firm. At the same time, each polluter has a horizontal incentive to be perceived as a low-cost firm by its rival, since this enables the latter to decrease its production in period 2 and the former to increase its output in the same period. Thus both incentives reinforce each other and, in the separating sequential equilibrium, lead the low-cost firms (those that produce and pollute a lot) to over-produce and over-pollute in the first period as compared to the non-signaling context with the aim of differentiating themselves from high-cost firms (those that produce and pollute a little). Consequently, high-cost polluters react by under-producing and under-emitting as compared to the benchmark case of non-signaling. The signaling process then causes a distortion that increases both the expected level of production and emissions in the industry. In order to reduce the upwards productive distortion of cost-efficient firms caused by signaling and to decrease aggregate production and emissions level, the policymaker imposes higher environmental taxes in the first period in the former context than in the latter context. This over-taxation leads in fact the firms that produce and pollute a lot to reduce their production and emissions levels in the first period, even though those of firms that produce and pollute very little increase.

Our findings also suggest that if the parameter valuing the environmental damage is sufficiently low, environmental taxes become negative (a subsidy per unit of pollution emitted), but the incentive of each polluter to persuade the other players that it is a low-cost firm persists. In this case, the policymaker's aim of increasing expected overall production requires an adjustment of firms' production levels. Specifically, in order to differentiate themselves from high-cost firms, low-cost firms need to increase their production and pollutant emitted, even though the less efficient firms reduce theirs. This is achieved by increasing the subsidy per unit of pollution in the signaling context as compared to the subsidy granted in the benchmark regime of non-signaling.

In our model, we also show that in the non-signaling regime the environmental tax is set by the policymaker below the marginal cost of environmental damages in order not to reduce the output further, but this is not necessarily the case in the signaling scenario. In this context, we find that the signaling effect, which offsets the distortion of under-production due to oligopolistic market power, leads the policymaker to set a pollution tax for each firm that, depending on the parameters of the model, may equal or even exceed the marginal environmental damage caused by each one.

The remainder of the paper comprises seven sections. Section 2 contains the model. Section 3 derives the equilibrium of the non-signaling game as well as the optimal period 1 and period 2 emissions taxes in such a game for the case in which the regulator's valuation of the environment is sufficiently high. Section 4 deals with the separating sequential equilibrium and the optimal environmental taxes in this equilibrium. In Section 5, the signaling outcome is compared with the non-signaling outcome. Section 6 compares the magnitude of pollution taxes with marginal environmental damages. Section 7 extends the analysis to the case in which the ecological conscience of the policymaker is sufficiently low to lead it to value production more than environment quality. Section 8 concludes.

2. Model set-up

Consider a single industry comprising two Cournot firms (i=A,B) that produce a homogeneous good through two production periods (t=1,2). For the sake of exposition, it is assumed that firms face, in each production period, the following inverse linear demand function[3]

$$p_t(Q_t) = 1 - Q_t\,, \tag{1}$$

p_t being the unit price of the good in period t when $Q_t = q_t^{\mathrm{A}} + q_t^{\mathrm{B}}$ units of output are sold in this period.[4]

The production process generates harmful emissions. We assume that each unit of output causes one unit of pollution emissions (Ulph, 1996; Bárcena and Garzón, 2002).[5] For simplicity, the environmental damage is assumed to be measured in each period t by the convex function:

$$ED_t(Q_t) = \frac{1}{2} d\, Q_t^2\,, \tag{2}$$

where $d > 0$ is an exogenous parameter that captures the policymaker's valuation of the environment or, put differently, the degree of ecological conscience.[6]

Each polluting firm can be of two possible types: low-cost or high-cost. Each definitely knows whether its marginal production cost is low or high, but does not have full knowledge of its rival's costs, only a prior belief (horizontal asymmetric information). With probability γ, it believes that the cost is low and with probability $1-\gamma$, it believes that the cost is high. The production cost of each polluting firm is also unobservable for the policymaker (vertical asymmetric information). With

[3] A more generalized form could be used, but calculating sequential equilibria would be more cumbersome.

[4] This demand is derived from the maximization problem of a representative consumer with utility separable in money, m_t, given by $U_t(Q_t) = u_t(Q_t) + m_t$, t=1,2, where $u_t(Q_t) = (1 - Q_t/2)\,Q_t$ is the utility function over the consumption good.

[5] Given this fixed relationship between production and pollution, an environmental tax is essentially equivalent to a production tax.

[6] Such a parameter may be also interpreted as the marginal willingness to pay for decreasing in one unit the environmental damage (see Bárcena and Garzón, 2002).

probability γ, it believes that the cost of each polluter is low, and with probability $1-\gamma$, it believes that it is high. In sum, the only common knowledge among players in period 1 is that the marginal cost of production of each firm *i* is constant and assumes either a low or a high value, each one being randomly selected. Specifically,

$$\tilde{c}^i = \begin{cases} 0 & \text{with probability } \gamma \\ c & \text{with probability } 1-\gamma, \end{cases} \tag{3}$$

where parameter *c* measures the efficiency gap between the “bad” and “good” realization of the cost of each firm and $\gamma \in (0,1)$. The bad outcome of $\tilde{c}^i$, *c*, is assumed to verify $0 < c < (1-d)/4$ given the inverse demand stated in (1) and the environmental damage function defined in (2).[7] The assumption that firms use the same technology leads them to emit the same pollution intensity. However, low-cost firms produce a lot of output and they also cause a higher emissions level than that caused by high-cost firms which produce less. Firms do not engage in emissions reductions activities.

Finally, it is assumed that the policymaker, who imposes a per-unit charge on pollution emissions in each production period, uses environmental taxes to control industry emissions. We treat the policymaker as an active player with full powers to set emissions taxes. Furthermore, it is assumed that all players, whether polluters or policymaker, are risk-neutral, and that the discount factor between periods normalized at one.

We regard the policymaker as seeking to maximize the per-period expected welfare function.[8] Such an objective function includes the unweighted sum of expected consumer surplus, CS_t, firms' expected profits, Π_t^i, *i*=A,B, and governmental expected revenue generated by pollution taxes, T_t, minus expected value of environmental damages caused by firms' production, ED_t (see Bárcena and Garzón, 2002; Okuguchi, 2003). That is,

[7] This upper bound in the bad realization of $\tilde{c}^i$ ensures that a high-cost firm *i* is always active in the product market whenever the realization of firm *j* and its beliefs (as well as those of the policymaker) about the type of *i* be.
[8] Administrative costs associated with environmental taxes are assumed to be negligible—an assumption that is in accordance with the conclusions drawn, for instance, for the Swedish Environment Protection Agency (SEPA). See SEPA (1997, p. 45).

$$W_t = CS_t + \Pi_t^{\mathrm{A}} + \Pi_t^{\mathrm{B}} + T_t - ED_t\,. \qquad (4)$$

The pollution-taxes game entails four stages. At the beginning of period 1 (first stage) and before observing the firms' output choice, the policymaker acts as a Stackelberg leader in setting environmental taxes for this period, e_1^i, i=A,B, to maximize social welfare in expected terms. The only common knowledge in this period is the distribution of the firms' costs. Given the prior probability assessment that each firm i has a certain cost efficiency and the environmental taxes chosen by the policymaker in period 1, each type of firm, acting as a Stackelberg follower, chooses its level of production so as to maximize its expected profits for period 1, q_1^i. It also generates a certain amount of pollution that affects the quality of environment. At the end of this period, the output of each firm i is observed publicly, from which the probability assessment of the competitor j and the policymaker regarding firm i's marginal cost is updated. Let $\gamma(q_1^i)$ be the common updated probability assessment as to the likelihood of the polluting firm i being a low-cost firm. Next, at the beginning of period 2 and given the updated beliefs both of firm j and the policymaker formed after observing firm i's output in period 1, the policymaker announces and commits, in the third stage, to a period 2 per-unit output-based environmental tax, e_2^i. Finally, in the fourth stage of the game, each polluter i chooses the profit-maximizing output for period 2, q_2^i, given the updated probability assessment $\gamma(q_1^i)$ and the environmental taxes e_2^i selected for this period.

The equilibrium concept we use for solving the proposed game is the sequential equilibrium (Kreps and Wilson, 1982), in which the period 1 outputs must constitute a Bayesian-Cournot equilibrium, the period 2 outputs must be chosen optimally given the updated probability assessments, and perceptions must satisfy the Bayes' rule (when it applies). In order to examine the role played by information transmission on optimal environmental taxation, two types of sequential equilibria are considered: separating and pooling equilibria.[9] Until Section 7, the analysis is restricted to the case in

[9] Since the paper focuses in examining the role that the signaling and the subsequent flow of information have on pollution taxes compared with the benchmark case of non-signaling, hybrid or semi-separating equilibria are not considered.

which the ecological conscience of the policymaker is sufficiently high that $d > 1/2$. Then, we also examine the case in which $0 < d < 1/2$.

3. Pooling equilibrium

Consider, as a benchmark case, the situation in which polluting firms do not signal their cost. Therefore, both types of polluters produce the same output in period 1, and, as a consequence, neither the policymaker nor the rival firm can update their priors about the efficiency level of the former. A pooling sequential equilibrium is, in this framework, a list of actions and beliefs, $\{e_1^i(\tilde{c}^i),\ q_1^i(\tilde{c}^i, e_1^i),\ \gamma(q_1^i(\cdot)),\ e_2^i(\tilde{c}),\ q_2^i(\tilde{c}, e_2)\}$ of the form

$$e_1^i(\tilde{c}^i) = e_1^i \text{, for all } \tilde{c}^i \in \{0, c\}\text{, } i\text{=A,B,} \tag{5}$$

$$q_1^i(\tilde{c}^i, e_1^i) = q_1^i \text{, for all } \tilde{c}^i \in \{0, c\}\text{,} \tag{6}$$

$$\gamma(q_1^i(\cdot)) = \gamma\text{,} \tag{7}$$

$$e_2^i(\tilde{c}^i) = e_2^{iP}\text{, for all } \tilde{c}^i \in \{0, c\}\text{,} \tag{8}$$

and

$$q_2^i(\tilde{c}^i, e_2^{iP}) = \begin{cases} q_{2L}^{iP}\text{, if } \tilde{c}^i = 0 \\ q_{2H}^{iP}\text{, if } \tilde{c}^i = c\text{,} \end{cases} \tag{9}$$

where superscript P stands for pooling equilibrium, and subscripts H and L denote, respectively, the high-cost and low-cost versions of firm i. As usual, such equilibrium is determined by using the classical backwards induction argument.

Period 2

Since the output of firms in period 1 does not signal their costs, the second period of the game is characterized by incomplete information. From the profit function of each polluting firm *i* in period 2,

$$\Pi_2^i = (1 - \tilde{c}^i - q_2^i - Eq_2^j)\, q_2^i - e_2^i q_2^i,\ i,j\text{=A,B};\ i \neq j,\ \text{and}\ \tilde{c}^i \in \{0, c\}, \tag{10}$$

where $Eq_2^j = \gamma q_{2L}^j + (1-\gamma) q_{2H}^j$, the output produced by high-cost firms is

$$q_{2H}^{iP} = \frac{2(1-c) - 2e_2^{iP} - \gamma c}{6}, \tag{11}$$

and that produced by low-cost firms is

$$q_{2L}^{iP} = \frac{2 - 2e_2^{iP} + (1-\gamma)c}{6}. \tag{12}$$

Thus, in order to find the optimal environmental taxes for the second-period game, the policymaker attempts to solve the following problem

$$\max_{e_2^{\mathrm{A}}, e_2^{\mathrm{B}}} W_2 = \frac{1}{2}(1-d)[\gamma(q_{2L}^{\mathrm{A}P} + q_{2L}^{\mathrm{B}P})^2 + (1-\gamma)(q_{2H}^{\mathrm{A}P} + q_{2H}^{\mathrm{B}P})^2] + \gamma[(q_{2L}^{\mathrm{A}P})^2 + (q_{2L}^{\mathrm{B}P})^2]$$

$$+ (1-\gamma)[(q_{2H}^{\mathrm{A}P})^2 + (q_{2H}^{\mathrm{B}P})^2] + e_2^{\mathrm{A}}[\gamma q_{2L}^{\mathrm{A}P} + (1-\gamma) q_{2H}^{\mathrm{A}P}] + e_2^{\mathrm{B}}[\gamma q_{2L}^{\mathrm{B}P} + (1-\gamma) q_{2H}^{\mathrm{B}P}], \tag{13}$$

and the first-order conditions lead to the optimal environmental taxes summarized in the following lemma.

Lemma 1. *The optimal pollution tax imposed on each firm in period 2 when firms' outputs do not signal firms' costs is* $e_2^{iP} = [(2d-1)/2(1+d)]\,[1-(1-\gamma)c]$, *i=A,B.*

In this case, the only effect the policymaker considers when deciding on the environmental taxes is the tradeoff between firms' under-production due to market power and firms' over-production due to harmful emissions. Since $d > 1/2$, the policymaker values environmental quality more than the effects of production both on consumer surplus, firms' profits and public revenue. Therefore, the optimal pollution tax imposed on each firm is positive. For a similar reasoning, the tax rate increases as parameters d and γ increase, but decreases as parameter c increases.

Substituting these pollution taxes into (11) and (12) yields the output and the amount of pollution

$$q_{2H}^{iP} = \frac{1 - c - \gamma dc}{2(1+d)}, \tag{14}$$

which are produced by each high-cost duopolist, and the output and the amount of pollution

$$q_{2L}^{iP} = \frac{1 + (1-\gamma)dc}{2(1+d)}, \tag{15}$$

which are produced by each low-cost duopolist.

4. Separating equilibrium

We now consider the situation in which the outputs produced by polluters in period 1 convey full information on their efficiency levels and we examine the optimal environmental taxation in this context. In this case, the period 2 game becomes a complete-information game in which both the policymaker and the rival firm j are fully informed about firm i's marginal cost. A separating sequential equilibrium is a set of actions and beliefs of the form $\{e_1^i(\tilde{c}^i), q_1^i(\tilde{c}^i, e_1^i), \gamma(q_1^i(\cdot)), e_2^i(\tilde{c}^i, q_1^i(\cdot)), q_2^i(\tilde{c}^i, e_2^i, q_1^i(\cdot))\}$ such as

$$e_1^i(\tilde{c}^i) = e_1^i \text{, for all } \tilde{c}^i \in \{0, c\},\ i\text{=A,B,} \tag{16}$$

$$q_1^i(\tilde{c}^i, e_1^i) = \begin{cases} q_{1H}^i, \text{ if } \tilde{c}^i = c \\ q_{1L}^i, \text{ if } \tilde{c}^i = 0, \end{cases} \tag{17}$$

$$\gamma(q_1^i(\cdot)) = \begin{cases} 0, \text{ if } q_1^i = q_{1H}^i \\ 1, \text{ if } q_1^i = q_{1L}^i, \end{cases} \tag{18}$$

$$e_2^i(q_1^i(\cdot), q_1^j(\cdot)) = \begin{cases} e_2^{iCI}(0,0), \text{if } q_1^i = q_{1L}^i \text{ and } q_1^j = q_{1L}^j \\ e_2^{iCI}(0,c), \text{if } q_1^i = q_{1L}^i \text{ and } q_1^j = q_{1H}^j \\ e_2^{iCI}(c,0), \text{if } q_1^i = q_{1H}^i \text{ and } q_1^j = q_{1L}^j \\ e_2^{iCI}(c,c), \text{if } q_1^i = q_{1H}^i \text{ and } q_1^j = q_{1H}^j \end{cases} \tag{19}$$

and

$$q_2^i(\tilde{c}^i, e_2^i(\cdot), q_1^i(\cdot)) = \begin{cases} q_{2L}^{iCI}(0,0), \text{if } \tilde{c}^i = 0 \text{ and } q_1^j = q_{1L}^j \\ q_{2L}^{iCI}(0,c), \text{if } \tilde{c}^i = 0 \text{ and } q_1^j = q_{1H}^j \\ q_{2H}^{iCI}(c,0), \text{if } \tilde{c}^i = c \text{ and } q_1^j = q_{1L}^j \\ q_{2H}^{iCI}(c,c), \text{if } \tilde{c}^i = c \text{ and } q_1^j = q_{1H}^j, \end{cases} \tag{20}$$

where superscript *CI* denotes a complete-information framework. That is, the policymaker selects period 1 environmental tax e_1^i for each polluting firm *i*, given the probability assessment that its cost is low, and each polluter *i* chooses output level q_1^i for period 1, given the environmental taxes to be paid. Next, for every q_1^i quoted, both the policymaker and the rival firm *j* update their beliefs about the cost level of firm *i* and the former also chooses the optimal environmental tax for period 2, e_2^i. Finally, the polluting firms select output levels q_2^i for the second production period. Updated beliefs $\gamma(q_1^i)$ are unrestricted, except that the Bayes' rule is used to establish them for actions with positive probability in equilibrium. As usual, a separating equilibrium as the one defined in (16)-(20) is derived by working backwards from the second period to the first.

Period 2

From the profit function of each polluter *i* in period 2, its optimal production level in this period is

$$q_2^i(\cdot) = \frac{1-2\tilde{c}^i - 2e_2^i + \tilde{c}^j + e_2^j}{3}, \quad i,j\text{=A,B; } i \neq j, \tag{21}$$

and the policymaker attempts to solve the problem:

$$\max_{e_2^i, e_2^j} W_2 = \frac{1}{2}(1-d)[q_2^i(\cdot)+q_2^j(\cdot)]^2 + [q_2^i(\cdot)]^2 + [q_2^j(\cdot)]^2 + e_2^i(\cdot)q_2^i(\cdot) + e_2^j(\cdot)q_2^j(\cdot), \tag{22}$$

which yields the environmental taxes recorded in the following lemma.

Lemma 2. *If* $d > 1/2$ *, the optimal tax imposed on each polluter i=A,B in period 2 is given by*

$$e_2^{iCI}(q_1^i(\cdot), q_1^j(\cdot)) = \begin{cases} (2d-1)/2(1+d), & \text{if } q_1^i = q_{1L}^i \text{ and } q_1^j = q_{1L}^j \\ 0, & \text{if } q_1^i = q_{1L}^i \text{ and } q_1^j = q_{1H}^j \\ [2d-1-(d-5)c]/(1+d), & \text{if } q_1^i = q_{1H}^i \text{ and } q_1^j = q_{1L}^j \\ (2d-1)(1-c)/2(1+d), & \text{if } q_1^i = q_{1H}^i \text{ and } q_1^j = q_{1H}^j. \end{cases}$$

Proof. See the Appendix.

While in the benchmark situation of non-signaling both polluting firms are taxed in period 2, regardless of their type, in a complete-information setting, this occurs only when firms have the same cost-efficiency. If their costs differ, the unique firms that are taxed are high-cost firms, i.e. those that produce and pollute very little. The intuition of this apparently striking result lies in the fact that the policymaker places a high value on environmental damage. Accordingly, with the aim of decreasing total environmental damage in this period, it taxes the high-cost polluter. This increases the asymmetry between the effective costs of firms and decreases competition in the marketplace so that the industry performance approximates that of a monopoly. In other words, if the firm causing the most emissions level were taxed rather than the firm that generates the less emissions level, the actual costs of both firms would converge and competition between them would be increased. Thus, both overall production and aggregate environmental damage would also increase.[10]

The second concern to be stressed in the light of Lemma 2 is that taxes paid by high-cost polluters are greater than those paid by low-cost firms. Once again, this is due to the fact that the policymaker tries to reduce overall emissions in the second period by acting strategically in response to the firms' behavior. In fact, increasing the effective cost gap between firms brings the industry behavior closer to monopoly behavior and reduces the emissions level in expected terms.

From (20) and taking into account Lemma 2, the output produced (and the pollutant emitted) by each polluter in period 2 is

$$q_2^{iCI}(\cdot) = \begin{cases} 1/2(1+d), & \text{if } \tilde{c}^i = 0, \text{and } q_1^j = q_{1L}^j \\ (d+2c)/(1+d), & \text{if } \tilde{c}^i = 0, \text{and } q_1^j = q_{1H}^j \\ (1-d-4c)/(1+d), & \text{if } \tilde{c}^i = c, \text{and } q_1^j = q_{1L}^j \\ (1-c)/2(1+d), & \text{if } \tilde{c}^i = c, \text{and } q_1^j = q_{1H}^j \end{cases} \tag{23}$$

and the fact that $\Pi_2^{iCI}(\tilde{c}^i, q_1^j(\cdot)) = [q_2^{iCI}(\cdot)]^2$ allows us to conclude (i) that a high-cost polluting firm earns lower profits in period 2 than a low-cost firm, which is due to the fact that each polluter pays a lower environmental tax in this period when the policymaker believes that it is of a low-cost type, and (ii) that each firm also earns more profits in period 2 when its rival is persuaded that it is a low-cost firm, since this induces the former to produce a lower level of output in such a period. Each polluter then has both a vertical and a horizontal incentive to be perceived as a low-cost firm. The following lemma records this result.

Lemma 3. *If $d > 1/2$, each polluting firm is interested in being perceived as a low-cost firm.*

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[10] If there were an international agency that could do the same as our policymaker at an international level, our findings could be understood as follows: under the assumption that the industry of all countries uses the same technology and they only differ in efficiency terms, the country owning the industry that produces less and pollutes less (the poor country) would bear the pollution taxes rather than the country with the industry that produces more and hence pollutes more too (the rich country).

Proof. See Appendix.

Each polluter i has the incentive to convince the policymaker that it is a low-cost firm (a firm that produces and pollutes a lot) because it leads the policymaker to tax it with a lower amount in such a period. At the same time, its incentive to convince the rival j that it is a low-cost firm derives from the fact that this leads firm j to react "softly" by reducing the production level in period 2 and, consequently, firm i to increase its production level in such a period. High-cost polluters then have a strong incentive to misrepresent themselves as low-cost firms by increasing their output levels in period 1 above the profit-maximizing levels of the one-shot incomplete-information game. This, in turn, leads low-cost firms, in order to distinguish themselves from high-cost ones, so as to increase their production above the level that maximizes their expected profits of the one-shot incomplete-information game.

Period 1

The following lemma establishes the outputs of polluters in period 1 that form part of the separating equilibrium.

Lemma 4. *If* $d > 1/2$ *, the output produced by each polluting firm i in period 1 that forms part of the separating equilibrium of minimum cost is* $q_{1L}^{i} = (1-e_1^i)/3 + [(3-\gamma)M^{1/2} - 2(1+d)c]/6(1+d)$ *, when it is a low-cost firm, and* $q_{1H}^{i} = (1-c-e_1^i)/3 - \gamma M^{1/2}/6(1+d)$ *, when it is of a high-cost type, where* $M = (d^2 - 6d + 8\gamma d - 71\gamma + 8)c^2 + 2(1+6d-2d^2+2d^2\gamma - 23d\gamma + 14\gamma)c + 4d^2 - 2(2d-1)^2\gamma - 1.$

Proof. See Appendix.

The lemma states that, in order to differentiate themselves from high-cost firms, the firms that produce and pollute a lot are obliged to produce, in period 1, a greater output than the maximizing-profit level of the one-shot incomplete-information game corresponding to this period, namely $q^i_{1L} > (1-e^i_1)/3 + \quad + (1-\gamma c)/6 \equiv q^{iII}_{1L}$. Furthermore, since firms' outputs are strategic substitutes, the best-response function of high-cost firms leads them to produce in period 1 a lower level than they would in order to maximize profits in the one-shot incomplete-information game, i.e. $q^i_{1H} < (1-c-e^i_1)/3 - \gamma/c \equiv q^{iII}_{1H}$. In sum, we derive a costly separating equilibrium in which signaling leads firms to produce a greater expected level output (and also to emit a higher expected level of pollution) than the level that maximizes profits in the incomplete-information period.

After determining the output levels that form part of the separating equilibrium, the policymaker sets the optimal environmental taxes in period 1. These taxes are summarized in the following lemma where superscript *S* stands for separating equilibrium.

Lemma 5. *If $d > 1/2$ and the firms' output in period 1 serves to signal firms' costs, the environmental tax in period 1 for each polluter i is $e^{iS}_1 = [(2d-1)/2(1+d)^2][(1-c)(1+d)+\gamma M^{1/2}]$, where M is defined as in Lemma 4.*

Proof. See Appendix.

In this case, the policymaker considers not only the tradeoff between production and pollution as in the benchmark case of non-signaling, but it also considers firms' behavior in their attempt of signaling. In particular, the fact that overall production and pollution emissions increase due to the increase in production of low-cost firms (those that produce a lot and also pollute a lot), even though those of high-cost firms decrease. Thus, the environmental tax imposed in this scenario is intended to reduce the output (and emissions) of low-cost firms and increase the output (and emissions) of high-cost firms. This will reduce the total environmental damage.

Taking into account Lemma 4, the quantity produced and the pollution level emitted by each polluter in period 1 is

$$q_{1L}^{i} = \frac{(1+d)(1-c)+(1+d-d\gamma)M^{1/2}}{2(1+d)^{2}} \tag{24}$$

in the case of a low-cost polluter, and

$$q_{1H}^{i} = \frac{(1+d)(1-c)-d\gamma M^{1/2}}{2(1+d)^{2}} \tag{25}$$

for a high-cost polluter.

5. Comparison between the signaling and non-signaling equilibria

In comparing the environmental taxes imposed in both regimes, the following result holds.

Proposition 1. *The optimal environmental tax required of to each polluter i is such that* $e_1^{iS} > e_2^{iP}$.

Proof. Straightforward from Lemmas 1 and 5. ■

From Lemma 4, the signaling effect leads to the total expected output (and emissions level) being increased, compared to the benchmark case of non-signaling. Moreover, the policymaker wishes to decrease industry emissions, which requires a decrease in total industry output. Thus, it is necessary to decrease the cost of signaling or, more specifically, that low-cost firms that produce more and then pollute more, decrease their production and polluting levels at equilibrium, even though high-cost firms that produce and pollute less increase theirs. The way to obtain these readjustments of production and emissions levels between firms is to increase the environmental tax imposed on each, above the amount that would prevail in the benchmark situation of non-signaling. Greater environmental taxes enable the policymaker to decrease the firms' deviation, in terms of production and emissions, from their optimal behavior in the one-shot incomplete-information game, since low-cost firms can signal themselves by producing an output level closer to their profit-maximizing output in the one-shot incomplete information game, q_{1L}^{iII}, and therefore, high-cost firms may produce an output level which is also closer to their optimal production in the one-shot incomplete-information game of the first period, q_{1H}^{iII}.[11]

From Proposition 1, two corollaries may be formulated.

Corollary 1. *The over-taxation that holds in the signaling context (as compared to the benchmark case of non-signaling) increases with parameters d and γ.*

Corollary 2. *If polluters signal their costs, then, in equilibrium, expected output, overall pollution level, and expected social welfare in period 1 are higher than in the benchmark case of non-signaling.*

6. Are environmental taxes always below marginal pollution damages?

It has generally been established that in the case of imperfectly competitive firms whose production harms the environment, there are two types of distortion relative to the social optimum; namely, (i) over-production due to emissions and (ii) under-production due to the exercise of market power. It is the tradeoff between these two misallocations that determines the magnitude of pollution taxes. A tax on pollution emissions reduces environmental damage, but it may also lead firms to reduce their output further, since it ignores the social cost of further output reduction by firms whose production level is already inefficient. Given that the market distortion cannot be corrected directly, the environmental tax can only achieve an optimal second-best in which it is set below the marginal value of environmental damages.[12] For an externality created by a monopolist, Buchanan (1969) and Barnett (1980) have shown in fact that the second-best taxing policy is to impose a tax below the marginal damage caused by pollution.

[11] Despite of total output and total emissions level are reduced from the action of increased environmental taxes, they are higher than in the non-signaling game. See Corollary 2.
[12] A solution to the problem would consist of managing two policy actions: a subsidy per unit of output to correct the product market distortion and a tax on emissions to correct the pollution externality. In this case, the tax would equal the marginal environmental damage, i.e. it would be the first-best tax (see Barnett, 1980; or Kennedy, 1994, among others).

This result is sustainable in an oligopolistic industry. In particular, the second-best emissions taxes are lower than the marginal damages in the context of a homogeneous good Cournot (open loop) oligopoly with a fixed number of equally efficient firms that do not engage in emissions reduction activities (Simpson, 1995; Lee, 1999).[13] Conversely, Simpson (1995) shows that the optimal tax is not necessarily lower than marginal environmental costs when firms are heterogeneous, since a Cournot duopoly might result in inefficient allocations of production between firms, and if a higher tax shifts production to the more efficient firm, then it might be optimal to set the tax higher. Carlsson (2000) extends the analysis to the Cournot-Nash closed loop and Stackelberg-Nash duopoly cases, and contemplates several forms of information transmission and how they affect optimal taxes. He concludes that whether the optimal tax is lower or higher than marginal environmental damage depends on the information transmission and the effect of the firms' strategic variable capital (e.g., investment in abatement capital) on their marginal costs.

But how do incomplete information and a dynamic context affect the relationship between optimal taxes and marginal environmental costs? The answer provided by our model depends on whether or not firms' private information is disclosed. In the benchmark case of non-signaling where the firms' information is not revealed, the policymaker considers the tradeoff between under-production due to market power and over-production due to emissions. In this context, pollution taxes are unambiguously set below marginal environmental damages in order to avoid an excessive decrease in expected output. However, this is not necessarily true in the signaling regime, in which case we must add the signaling effect to the above effects, in such a way that the emission tax may be lower, equal to or even higher than the marginal cost of damages. Formally,

Proposition 2. *In the signaling game, the (second-best) pollution taxes are higher (equal, lower) than marginal environmental damages if parameters d, c, and γ satisfy* $[2d(1-\gamma)-1](1-c)(1-d)/2(1+d)-[1-d(2-3d(1-\gamma)-\gamma)]\gamma M^{1/2}/2(1+d)^2>0$ *(=, <).*

[13] Poyago-Theotoky (2003) examines the case in which polluting firms compete in Cournot or Bertrand fashion, produce differentiated goods and the regulator has two policy tools (the emissions tax and the subsidy on firms' R&D, which reduces emissions levels), and yields a similar result.

This result, which is obtained by comparing environmental tax e_1^{iS} and marginal cost of environmental damage $MC = d[\gamma q_{1L}^{iS} + (1-\gamma) q_{1H}^{iS}]$, is illustrated in Fig. 1 for the case in which $d = 3/4$.

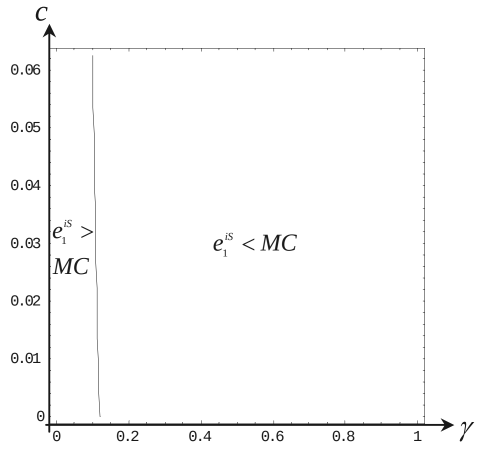

Fig. 1. Pollution taxes in the signaling context as compared to the marginal environmental damage.

The explanation of the first part of Proposition 2 is as follows. If the probability of polluters being low-cost firms is sufficiently low, under-production relative to the output of the one-shot incomplete-information game of the first period (which is due to the signaling effect) is minimal. Given that the emission tax is geared towards correcting the pollution externality and that the expected output (and hence expected emissions) increases as the probability of polluters being low-cost firms decrease, the optimal tax is set above the marginal environmental costs to avoid an excessive increase in output (and pollutant emitted). The other two claims of the proposition are similarly explained.

7. The case of a low ecological conscience

In this section, we examine the situation in which the ecological conscience of the policymaker is sufficiently low that $0 < d < 1/2$. In this case, the bad realization of the cost level of each polluter needs to be restricted to values satisfying $c \in (0, d/(2+d))$ to ensure that a high-cost polluting firm always finds profitable to produce a positive output in period 2 regardless of the rival's cost.

From Lemma 1, it can easily be demonstrated that there is a per-unit subsidy in the benchmark non-signaling context, $e_2^{iP} < 0$. This is due to the fact that production is valued higher than environmental damage and firms are subsidized to offset the effect of market power on output and to increase overall industry production. In this setting, the quantities produced in each period are those given in (14) and (15). That is, as compared to the case in which $d > 1/2$, production of high-cost firms which produce a little decreases and that of low-cost firms which produce a lot increases. Naturally, this leads to a higher expected level of emissions.

Conversely, in the scenario in which firms' outputs in period 1 serve to obtain information about firms' costs, the following occurs in the second period:

Lemma 6. *If $d < 1/2$, and firms' output signal their costs, the environmental tax to be paid by each polluter i=A,B in period 2 is negative (a subsidy per unit of pollution emitted) and it amounts to*

$$e_2^{iCI}(q_1^i(\cdot), q_1^j(\cdot)) = \begin{cases} (2d-1)/2(1+d), & \textit{if } q_1^i = q_{1L}^i \textit{ and } q_1^j = q_{1L}^j \\ [2d-1-(4+d)c]/(1+d), & \textit{if } q_1^i = q_{1L}^i \textit{ and } q_1^j = q_{1H}^j \\ 0, & \textit{if } q_1^i = q_{1H}^i \textit{ and } q_1^j = q_{1L}^j \\ (2d-1)(1-c)/2(1+d), & \textit{if } q_1^i = q_{1H}^i \textit{ and } q_1^j = q_{1H}^j. \end{cases}$$

Proof. See Appendix.

In contrast to the case in which $d > 1/2$, here, the policymaker is more concerned about production than environment quality. Therefore, the subsidy received by low-cost firms which produce a lot, and then cause more pollution, exceeds that received by high-cost firms. Furthermore, when both polluters are not equally efficient, only that which produces more and also pollutes more is granted a subsidy in period 2. The firms that produce and pollute less do not receive a subsidy in such period.

Taking into account both the firms' maximized profits in period 2,

$$\Pi_2^{iCI}(\tilde{c}^i, \tilde{c}^j) = \begin{cases} [1/2(1+d)]^2, & \text{if } q_1^i = q_{1L}^i \text{ and } q_1^j = q_{1L}^j \\ \{[1-d+(3+d)c]/(1+d)\}^2, & \text{if } q_1^i = q_{1L}^i \text{ and } q_1^j = q_{1H}^j \\ \{[d-(2+d)c]/(1+d)\}^2, & \text{if } q_1^i = q_{1H}^i \text{ and } q_1^j = q_{1L}^j \\ [(1-c)/2(1+d)]^2, & \text{if } q_1^i = q_{1H}^i \text{ and } q_1^j = q_{1H}^j, \end{cases} \tag{26}$$

and the profits they would obtain in the off-the-equilibrium path, we obtain the following result.

Lemma 7. *If* $d < 1/2$*, each polluter wishes to be perceived as a low-cost firm.*

Proof. See Appendix.

In the light of both Lemma 7 and Lemma 2, it follows that each polluting firm has an incentive to be regarded as a low-cost firm which produces a lot and also pollutes a lot, irrespective of the value of the parameter measuring the ecological conscience of the policymaker is low or high. This conclusion is in stark contrast with that achieved when there is a single polluter, in which case the preference of the monopolistic firm to be perceived as a low-cost or a high-cost firm depends on whether the value of the parameter measuring the ecological conscience of the policymaker is sufficiently high or low.[14]

The next lemma describes the outputs that form part of the separating equilibrium of minimum cost.

Lemma 8. *If* $d < 1/2$*, the output produced by each polluter i in period 1 that forms part of the separating equilibrium is* $q_{1L}^{i} = (1-e_1^i)/3 + [(3-\gamma)R^{1/2} - 2(1+d)c]/6(1+d)$*, when the polluter is a low-cost type, and* $q_{1H}^{i} = (1-c-e_1^i)/3 - \gamma R^{1/2}/6(1+d)$*, when it is a high-cost type, where*

$$R = [24 + d(10+d) - 39\gamma - 4d(6+d)\gamma]c^2 + 2[11 - d(8 + d(2-6\gamma) - 15\gamma) - 12\gamma]c +$$
$$+ (1-2d)[3 - 2\gamma + d(-2+4\gamma)].$$

Proof. See Appendix.

Derived from the signaling effect, low-cost firms produce, in the separating equilibrium of minimum cost, a greater output level than in the one-shot incomplete-information game of period 1,

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[14] See Antelo (2005).

and high-cost firms produce a lower amount (and causes a lower emissions level) than in the one-shot incomplete-information game of period 1. From this firms' behavior, the policymaker imposes, in period 1, the following tax rate on each firm.

Lemma 9. *If* $d < 1/2$ *and the output produced by each polluter in the first period serves to provide information about its costs, the policymaker sets the environmental subsidy* $e_1^{iS} = (2d-1)[(1-c)(1+d)+\gamma R^{1/2}]/2(1+d)^2$ *, where R is defined as in Lemma 8.*

Proof. See Appendix.

The policymaker is now more concerned about production than about environment quality, such that environmental taxes are also negative in this signaling context. Moreover, the level of these subsidies results from the interaction of two forces: the tradeoff between production and emissions and the firms' deviation in production, and pollution, behavior during period 1 (from their optimal behavior in an incomplete-information context) to make signaling possible.

Turning again to Lemma 8, the quantity produced and the pollutant emitted by each low-cost polluter in the first period is

$$q_{1L}^i = \frac{(1+d)(1-c)+(1+d-d\gamma)R^{1/2}}{2(1+d)^2}, \tag{27}$$

and that produced by each high-cost polluting firm is

$$q_{1H}^i = \frac{(1+d)(1-c)-d\gamma R^{1/2}}{2(1+d)^2}. \tag{28}$$

Finally, comparing the level of subsidies in the separating and pooling equilibria allows us to obtain the following result.

Proposition 3. *When* $d < 1/2$, *the per-unit environmental subsidy to each polluter i=A,B in the signaling context is higher than in the benchmark case of non-signaling; namely,* $\left|e_1^{iS}\right| > \left|e_2^{iP}\right|$.

The explanation of the proposition is as follows. The signaling effect increases total expected output and emissions. In addition, the policymaker wishes to increase the overall level of output, which requires increasing the cost of signaling, i.e. increasing the production level of low-cost firms (above their optimal production in a non-signaling context, q_{1L}^{iII}), even though the production of high-cost firms decreases with respect to their optimal production in a non-signaling situation, q_{1H}^{iII} . In order to do this, the policymaker sets a greater subsidy per unit of emissions in the signaling context than in the benchmark case, in which firms' quantities do not signal firms' costs.

Corollary 3. *Over-subsiding in the signaling context (as compared to the benchmark case of non-signaling) increases as parameter d decreases and parameter* γ *increases.*

Corollary 4. *If* $d < 1/2$, *expected output, pollution level, and expected social welfare in period 1 are higher when polluters signal their costs than in the benchmark case of non-signaling.*

8. Concluding remarks

The analysis of pollution taxes and charges as well as other implemented market-based instruments to control pollution emissions, is relevant not only for theoretical environmental economics, but also for political discourse and debate, especially when considering whether or not such economic instruments should be introduced in the future and to what extent. In this paper, we have examined how the extent to which firms can fully reveal their pollution levels affects the optimal level of environmental taxation. The assumption that there is confidential information which is disclosed complicates the analysis of pollution taxes, since the policymaker must consider the signaling effects when choosing appropriate tax rates. However, this is a relevant issue, because it reflects a plausible fact in real-life industries.

Our findings suggest that the policymaker uses environmental taxes to change firms' behavior. In particular, if there are several Cournot polluters producing a homogenous good, both horizontal and vertical asymmetric information prevail, and the ecological conscience of the policymaker is sufficiently high, then the optimal environmental taxes when the firms' output serves to provide information about their costs are higher than they would be if firms' costs were not signaled**.** This over-taxation in the signaling context allows the policymaker to decrease overall emissions through decreasing the signaling cost in terms of firms' productive deviation: low-cost firms which produce and pollute more decrease their output and emissions levels (and these approach the levels of the non-signaling context) and high-cost firms which produce and pollute a little increase their levels (and they also move in the direction of the levels that would prevail in the absence of signaling).

By contrast, when the policymaker values production more than the quality of environment, it seeks to maximize industry production and this leads environmental taxes to become negative (a subsidy per unit of emissions). In the signaling context, the aim of the policymaker is achieved by setting an environmental subsidy in the first period above the level that would prevail in the case of non-signaling. This increases the signaling cost (in production terms) or, more precisely, readjusts the production levels of polluters in such a way that low-cost firms find it more difficult to distinguish themselves from high-cost firms and, consequently, they need to increase their production level (and emissions).

Appendix

Proof of Lemma 2. Given the profit function of each polluter in the second period (see (10)) and the output level (21) that maximizes expected profits, the policymaker solves the problem

$$\max_{e_2^{\mathrm{A}},e_2^{\mathrm{B}}} W_2 = \frac{1}{2}(1-d)\left(\frac{2-\tilde{c}^{\mathrm{A}}-\tilde{c}^{\mathrm{B}}-e_2^{\mathrm{A}}-e_2^{\mathrm{B}}}{3}\right)^2+\left(\frac{1-2\tilde{c}^{\mathrm{A}}-2e_2^{\mathrm{A}}+\tilde{c}^{\mathrm{B}}+e_2^{\mathrm{B}}}{3}\right)^2$$

$$+\left(\frac{1-2\tilde{c}^{\mathrm{B}}-2e_2^{\mathrm{B}}+\tilde{c}^{\mathrm{A}}+e_2^{\mathrm{A}}}{3}\right)^2+e_2^{\mathrm{A}}\frac{1-2\tilde{c}^{\mathrm{A}}-2e_2^{\mathrm{A}}+\tilde{c}^{\mathrm{B}}+e_2^{\mathrm{B}}}{3}+e_2^{\mathrm{B}}\frac{1-2\tilde{c}^{\mathrm{B}}-2e_2^{\mathrm{B}}+\tilde{c}^{\mathrm{A}}+e_2^{\mathrm{A}}}{3} \quad \text{(A1)}$$

in order to find the optimal environmental taxes for period 2. When both polluters have the same marginal cost ($\tilde{c}=0$ or $\tilde{c}=c$), the problem (A1) has an interior solution given by $e_2^{\mathrm{A}}=e_2^{\mathrm{B}}=(2d-1)(1-\tilde{c})/2(1+d)$. However, when their costs are different, an interior solution does not exist for the problem stated in (A1). Instead, it has a corner solution meaning that the policymaker only taxes one of the two firms and not all of them. In particular, if only the low-cost polluter (say, firm A) is taxed ($e_2^{\mathrm{A}}>0,\ e_2^{\mathrm{B}}=0$), then the equilibrium outputs for period 2 are $q_2^{\mathrm{A}}=(1+c-2e_2^{\mathrm{A}})/3$ and $q_2^{\mathrm{B}}=(1-2c+e_2^{\mathrm{A}})/3$, and the policymaker's problem

$$\max_{e_2^{\mathrm{A}}} W_2=\frac{1}{2}(1-d)\left(\frac{2-c-e_2^{\mathrm{A}}}{3}\right)^2+\left(\frac{1+c-2e_2^{\mathrm{A}}}{3}\right)^2+\left(\frac{1-2c+e_2^{\mathrm{A}}}{3}\right)^2+e^{\mathrm{A}}\left(\frac{1+c-2e_2^{\mathrm{A}}}{3}\right) \quad \text{(A2)}$$

yields the solution $e_2^{\mathrm{A}}=[2d-1-(d+4)c]/(1+d)$. Conversely, if only the high-cost polluter (say, firm A) is taxed, then the output levels produced at equilibrium in period 2 are $q_2^{\mathrm{A}}=(1-2c-2e_2^{\mathrm{A}})/3$ and $q_2^{\mathrm{B}}=(1+c+e_2^{\mathrm{A}})/3$, and the policymaker's problem,

$$\max_{e_2^{\mathrm{A}}} W_2=\frac{1}{2}(1-d)\left(\frac{2-c-e_2^{\mathrm{A}}}{3}\right)^2+\left(\frac{1-2c-2e_2^{\mathrm{A}}}{3}\right)^2+\left(\frac{1+c+e_2^{\mathrm{A}}}{3}\right)^2+e^{\mathrm{A}}\left(\frac{1-2c-2e_2^{\mathrm{A}}}{3}\right), \quad \text{(A3)}$$

leads to the optimal environmental tax $e_2^{\mathrm{A}}=[2d-1-(d-5)c]/(1+d)$. Finally, by evaluating expected social welfare at both tax levels, it follows that $W_2\big|_{e_2^{\mathrm{A}}=[2d-1-(d+4)c]/(1+d)}-W_2\big|_{e_2^{\mathrm{A}}=[2d-1-(d-5)c]/(1+d)}<0$ (>) if $d>1/2$ (<). ■

Proof of Lemma 3. Denote by $\Pi_2^i(0|c,\tilde{c}^j)$ the maximized-profit of each polluter *i* in period 2 when it is a low-cost firm, but both the competitor *j* and the policymaker believe it to be high-cost, and it is common knowledge that $\tilde{c}^j = 0$ with probability γ and $\tilde{c}^j = c$ with probability $1-\gamma$. Similarly, denote by $\Pi_2^i(c|0,\tilde{c}^j)$ the maximized-profit of the polluting firm *i* in period 2 when it is a high-cost firm, but both the rival firm *j* and the policymaker believe it is a low-cost firm, whereas firm *j* is either a low-cost or a high-cost firm. Computing such profits leads to

$$\begin{aligned}\Pi_2^i(0|c,0) &= \left[\frac{2(1-d)-(7-d)c}{2(1+d)}\right]^2 \\ &= \left[q_2^i(0,0)+\frac{1-2d-(7-d)c}{2(1+d)}\right]^2,\end{aligned} \tag{A4}$$

$$\begin{aligned}\Pi_2^i(0|c,c) &= \left[\frac{1+dc}{2(1+d)}\right]^2 \\ &= \left[q_2^i(0,c)+\frac{1-2d-(4-d)c}{2(1+d)}\right]^2,\end{aligned} \tag{A5}$$

$$\begin{aligned}\Pi_2^i(c|0,0) &= \left[\frac{1-(1+d)c}{2(1+d)}\right]^2 \\ &= \left[q_2^i(c,0)-\frac{1-2d-(7-d)c}{2(1+d)}\right]^2,\end{aligned} \tag{A6}$$

and

$$\begin{aligned}\Pi_2^i(c|0,c) &= \left[\frac{2d+(3-d)c}{2(1+d)}\right]^2 \\ &= \left[q_2^i(c,c)-\frac{1-2d-(4-d)c}{2(1+d)}\right]^2.\end{aligned} \tag{A7}$$

Finally, comparing equilibrium profits defined by $\Pi_2^i(\tilde{c}^i,\tilde{c}^j) = [q_2^{iCI}(\cdot)]^2$, where $q_2^{iCI}(\cdot)$ is the output level of each firm defined in (21), and out-of-equilibrium profits given in (A4)-(A7) yields

$\Pi_2^i(0|c,0) < \Pi_2^i(0,0)$, $\Pi_2^i(c|0,0) > \Pi_2^i(c,0)$, $\Pi_2^i(0|c,c) < \Pi_2^i(0,c)$, and $\Pi_2^i(c|0,c) > \Pi_2^i(c,c)$, since $1-2d-(4-d)c<0$. This concludes the proof of the lemma. ■

Proof of Lemma 4. The incentive compatibility conditions for a separating equilibrium to prevail are

$$q_{1H}^i(q_{1L}^j, q_{1H}^j, e_1^i) = \underset{q_{1H}^i}{\arg\max}\,[\gamma \Pi_{1H}^i(q_{1H}^i, q_{1L}^j, e_1^i) + (1-\gamma)\Pi_{1H}^i(q_{1H}^i, q_{1H}^j, e_1^i)], \tag{A8}$$

$$\Pi_{1L}^i(q_{1L}^i, \tilde{q}_1^j, e_1^i) + \Pi_{2L}^i(0, \tilde{c}^j) \geq \Pi_{1L}^i(q_{1H}^i, \tilde{q}_1^j, e_1^i) + \Pi_{2L}^i(0|c, \tilde{c}^j), \tag{A9}$$

and

$$\Pi_{1H}^i(q_{1L}^i, \tilde{q}_1^j, e_1^i) + \Pi_{2H}^i(c|0, \tilde{c}^j) \leq \Pi_{1H}^i(q_{1H}^i, \tilde{q}_1^j, e_1^i) + \Pi_{2H}^i(c, \tilde{c}^j), \tag{A10}$$

where condition (A8) yields the profit-maximizing output of a high-cost polluter in the one-shot incomplete-information game of period 1, condition (A9) is the incentive compatibility condition for each low-cost polluter, and condition (A10) is the self-selection constraint for each high-cost polluter. Particularizing (A8)-(A10) and deleting superscripts to simplify the notation, we have, respectively,

$$q_{1H}^i \equiv f(q_{1L}^i) = \frac{1-c-e_1^i-\gamma q_{1L}^i}{3-\gamma}, \tag{A8a}$$

$$\begin{aligned}
&\gamma(1-e_1^i-2q_{1L}^i)\, q_{1L}^i + (1-\gamma)[1-e_1^i-q_{1L}^i - f(q_{1L}^i)]\, q_{1L}^i + \gamma\left[\frac{1}{2(1+d)}\right]^2 + (1-\gamma)\left(\frac{2c+d}{1+d}\right)^2 \\
&\quad \geq \gamma[1-e_1^i-h(q_{1L}^i)-q_{1L}^i]\, h(q_{1L}^i) + (1-\gamma)[1-e_1^i-h(q_{1L}^i)-f(q_{1L}^i)]\, h(q_{1L}^i) \\
&\quad + \gamma\left[\frac{2(1-d)-(7-d)c}{2(1+d)}\right]^2 + (1-\gamma)\left[\frac{1+dc}{2(1+d)}\right]^2,
\end{aligned} \tag{A9a}$$

and

$$\gamma(1-c-e_1^i-2q_{1L}^i)\,q_{1L}^i+(1-\gamma)[1-c-e_1^i-q_{1L}^i-f(q_{1L}^i)]\,q_{1L}^i+\gamma\left[\frac{1-(1+d)c}{2(d+1)}\right]^2$$

$$+(1-\gamma)\left[\frac{2d+(3-d)c}{2(1+d)}\right]^2\leq\gamma[1-c-e_1^i-f(q_{1L}^i)-q_{1L}^i]\,f(q_{1L}^i)+(1-\gamma)[1-c-e_1^i-2f(q_{1L}^i)]\,f(q_{1L}^i)$$

$$+\gamma\left(\frac{1-d-4c}{1+d}\right)^2+(1-\gamma)\left[\frac{1-c}{2(1+d)}\right]^2 \qquad \text{(A10a)}$$

where $h(q_{1L}^i)=[2+(1-\gamma)c-2e_1^i-2\gamma q_{1L}^i]/2(3-\gamma)$ is the best-response of a low-cost polluting firm competing with a rival which is low-cost with probability γ and, consequently, produces output q_{1L}^i and is high-cost with probability $1-\gamma$ and thus produces output $q_{1H}^i\equiv f(q_{1L}^i)$. The second-degree equation formed by taking (A9a) as implying equality has the roots

$$q_{1L}^i=\frac{2+(1-\gamma)c-2e_1^i}{6}\pm\frac{(3-\gamma)L^{1/2}}{6(1+d)}\ , \qquad \text{(A11)}$$

where $L=(-d^2+14\gamma d-65\gamma+16)c^2+2(7d+14\gamma-23d\gamma+2d^2\gamma)c+4d^2-2(2d-1)^2\gamma-1$, and the second-degree equation emerged from taking the self-selection condition (A10a) as implying equality, has the roots

$$q_{1L}^i=\frac{1-c-e_1^i}{3}\pm\frac{(3-\gamma)M^{1/2}}{6(1+d)}\,, \qquad \text{(A12)}$$

where $M=(d^2-6d+8\gamma d-71\gamma+8)c^2+2(1+6d-2d^2+2d^2\gamma-23d\gamma+14\gamma)c+4d^2-$

$-2(2d-1)^2\gamma-1$. Denoting by l^+ the highest root of the two given in (A11) and by m^+ the highest root of the two stated in (A12), it is not difficult to determine that, for values of parameters *d*, *γ*, and *c* such that $d > 1/2$, $\gamma \in (0,1)$, and $c < (1-d)/4$, the interval $[m^+, l^+]$ is non-degenerated in the sense that $m^+ < l^+$. Comparing the output level m^+ with the profit-maximizing output of each low-cost polluting firm as a mere duopolist in an incomplete-information setting, $q_{1L}^{iII} = [2 + (1-\gamma)c - 2e_1^i]/6$, it follows that $m^+ > q_{1L}^{iII}$. A separating equilibrium is then possible and it has the property that output q_{1L}^{iII} from each low-cost firm is not high enough for it to be able to distinguish itself from a high-cost polluter. Consequently, each low-cost firm is obliged to produce, in the separating equilibrium of minimum cost, an output level m^+ strictly greater than q_{1L}^{iII}, and each high-cost polluting firm reacts by producing output $q_{1H}^i = f(m^+)$ according to (A8a). ■

Proof of Lemma 5. The policymaker seeks to solve the problem

$$\max_{e_1^{\mathrm{A}}, e_1^{\mathrm{B}}} W_1 = \frac{1}{2}(1-d)[\gamma(q_{1L}^{\mathrm{A}} + q_{1L}^{\mathrm{B}})^2 + (1-\gamma)(q_{1H}^{\mathrm{A}} + q_{1H}^{\mathrm{B}})^2] + \gamma[(q_{1L}^{\mathrm{A}})^2 + (q_{1L}^{\mathrm{B}})^2]$$

$$+ (1-\gamma)[(q_{1H}^{\mathrm{A}})^2 + (q_{1H}^{\mathrm{B}})^2] + e_1^{\mathrm{A}}[\gamma q_{1L}^{\mathrm{A}} + (1-\gamma)q_{1H}^{\mathrm{A}}] + e_1^{\mathrm{B}}[\gamma q_{1L}^{\mathrm{B}} + (1-\gamma)q_{1H}^{\mathrm{B}}], \quad \text{(A13)}$$

where q_{1L}^i and q_{1H}^i are the output levels defined in Lemma 3. The first-order condition of problem (A13) and consideration of the fact that $e_1^{\mathrm{A}} = e_1^{\mathrm{B}} = e_1^{iS}$ yield the result claimed in the lemma. ■

Proof of Lemma 6. If $d < 1/2$, environmental taxes become negative, i.e. a subsidy per unit of pollutant emitted, and the objective function of the problem defined in (A2) when firms have different costs is maximized when only the low-cost firm which produces and pollutes more is subsidized. Consequently, the optimal environmental subsidies are those stated in the lemma. ■

Proof of Lemma 7. Denote by $\Pi_2^i(0|c, \tilde{c}^j)$ the maximized-profit of each polluter *i* in period 2 when its cost is low, but both the polluting firm *j* and the policymaker believe it is high, and it is common knowledge that the cost realization of polluting firm *j* is either low or high. Similarly, denote by $\Pi_2^i(c|0, \tilde{c}^j)$ the maximized-profit of the polluter *i* in period 2 when it is a high-cost firm, but both firm

j and the policymaker believe it is a low-cost type, whereas firm j is either low-cost or high-cost. A computation of such profits leads to

$$\Pi_2^i(0|c,0) = \left[\frac{2d-(3+d)c}{2(1+d)}\right]^2 \tag{A14}$$

$$\Pi_2^i(0|c,c) = \left[\frac{1+dc}{2(1+d)}\right]^2, \tag{A15}$$

$$\Pi_2^i(c|0,0) = \left[\frac{1-(1+d)c}{2(1+d)}\right]^2, \tag{A16}$$

and

$$\Pi_2^i(c|0,c) = \left[\frac{2(1-d)+(5+d)c}{2(1+d)}\right]^2 . \tag{A17}$$

Finally, a comparison of equilibrium profits stated in (26) and profits given in (A14)-(A17) yields $\Pi_2^i(0|c,0) < \Pi_2^i(0,0)$, $\Pi_2^i(0|c,c) < \Pi_2^i(0,c)$, $\Pi_2^i(c|0,0) > \Pi_2^i(c,0)$, and $\Pi_2^i(c|0,c) > \Pi_2^i(c,c)$. ■

Proof of Lemma 8. In this case, the incentive compatibility conditions for a separating equilibrium prevails as given in (A8), (A9), and (A10) are particularized in (A8a),

$$\begin{aligned}
&\gamma(1-e_1^i-2q_{1L}^i)\, q_{1L}^i + (1-\gamma)[1-e_1^i-q_{1L}^i - f(q_{1L}^i)]\, q_{1L}^i + \gamma\left[\frac{1}{2(1+d)}\right]^2 + (1-\gamma)\left[\frac{1-d+(3+d)c}{1+d}\right]^2 \\
&\geq \gamma[1-e_1^i-h(q_{1L}^i)-q_{1L}^i]\, h(q_{1L}^i) + (1-\gamma)[1-e_1^i-h(q_{1L}^i)-f(q_{1L}^i)]\, h(q_{1L}^i) \\
&+\gamma\left[\frac{2d-(3+d)c}{2(1+d)}\right]^2 + (1-\gamma)\left[\frac{1+dc}{2(1+d)}\right]^2,
\end{aligned} \tag{A9b}$$

and

$$\gamma(1-c-e_1^i-2q_{1L}^i)\, q_{1L}^i+(1-\gamma)[1-c-e_1^i-q_{1L}^i-f(q_{1L}^i)]\, q_{1L}^i+\gamma\left[\frac{1-(1+d)c}{2(1+d)}\right]^2$$

$$+(1-\gamma)\left[\frac{2(1-d)+(5+d)c}{2(1+d)}\right]^2$$

$$\le \gamma[1-c-e_1^i-f(q_{1L}^i)-q_{1L}^i]\, f(q_{1L}^i)+(1-\gamma)[1-c-e_1^i-2f(q_{1L}^i)]\, f(q_{1L}^i)$$

$$+\gamma\left[\frac{d-(2+d)c}{1+d}\right]^2+(1-\gamma)\left[\frac{1-c}{2(1+d)}\right]^2, \qquad \text{(A10b)}$$

respectively. The second-degree equation formed by taking (A9b) with equality has the roots

$$q_{1L}^i=\frac{2+(1-\gamma)c-2e_1^i}{6}\pm\frac{(3-\gamma)S^{1/2}}{6(1+d)}\ , \qquad \text{(A18)}$$

where $S=[3(2+d)(6+d)-(45+30d+4d^2)\gamma]c^2-\ 2[12(\gamma-1)+d(9+4d-15\gamma-6d\gamma)]c+$ $+(1-2d)(3-2d-2\gamma+4d\gamma)$ is positive, and the second-degree equation which emerges from taking the self-selection condition defined in (A10b) as equality has the roots

$$q_{1L}^i=\frac{1-c-e_1^i}{3}\pm\frac{(3-\gamma)R^{1/2}}{6(1+d)}, \qquad \text{(A19)}$$

where $R=[24+d(10+d)-39\gamma-4d(6+d)\gamma)]c^2+2[11-d(8+d(2-6\gamma)-15\gamma)-12\gamma]c+$ $+(1-2d)[3-2\gamma-d(2-4\gamma)]$ is positive. Denoting by s^+ the highest root of the two given in (A18) and by r^+ the highest root of the two given in (A19), it is simple to establish that the interval $[r^+,s^+]$ is non-degenerated for all values of parameters such that $d<1/2$, $\gamma\in(0,1)$ and $c\in(0,2/(d+2))$ for which reason a continuum of separating equilibria prevails. Comparing the quantity r^+ with the profit-maximizing output of each low-cost polluter as a simple duopolist under incomplete information, $q_{1L}^{iII}=[2+(1-\gamma)c-2e_1^i]/6$, it follows that $r^+>q_{1L}^{iII}$. Therefore, the separating equilibrium of minimum cost involves period 1 outputs in which production q_{1L}^{iII} is not high enough for low-cost firms to distinguish themselves from high-cost firms. Indeed, each low-cost firm is obliged to produce, in the first period, an output level strictly higher than q_{1L}^{iII}, and, consequently, each high-cost polluter produces an output strictly lower than q_{1H}^{iII}. This concludes the proof of the lemma. ■

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